COSC 2011
Section N

Instructor: Bill Kapralos

Tuesday, March 13 2001

Overview

- Containers
  - Inspectable Containers
- Abstract Data Types
  - Trees, Binary Trees
- Pseudo-Code
- Array Based Implementations
  - Stack
  - Queue
- Loop Invariants (Time Permitting)

Containers (1)

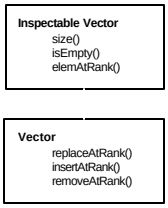
- Definition
  - A data structure that stores an organizes a collection of objects called the elements of the container
  - Provides access to them through the methods of the abstract data type.
  - Examples include: stacks, queues, dequeues, vectors, lists, sequences etc.
- Four main Categories for Methods of a Container
  - **Query Methods**: Return info. on the container or specific elements.
  - **Accessor Methods**: Return elements or positions of the container
  - **Update Methods**: Change the container by adding or removing elements or altering the relation between elements.
  - **Constructor Methods**: Generate an instance of the container.

Containers (2)

- Inspectable Containers
  - Containers that do not provide update methods
  - Support only read-only access and cannot be modified
  - Protect their elements from erroneous or malicious update attempts by other objects
  - Cannot be used when their elements are subject to updates during the life of their container.
  - Using inheritance we can get protection of inspect able containers while still getting flexibility of update methods

Containers (3)

- InspectableVector that contains only the query methods and the accessor method
- Redefine a Vector as an ADT that inspects the inspectableVector ADT and adds the update methods



- Revised vector is equivalent to the original one.
- But now we can reference an instance of the vector with a variable of type InspectableVector thus allowing only the query and accessor methods
- Can also restructure the List and Sequence ADTs by introducing inspect able versions of them.

Trees: Terminology and Basic Properties

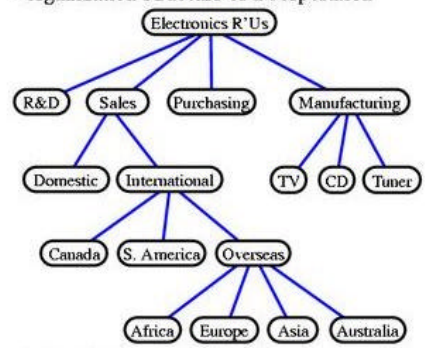
- Tree Abstract Data Type
  - Stores elements hierarchically
  - A set of nodes storing elements in a **parent-child** relationship.
  - Top element **r** is called the ***root***.
  - Each node **v** of tree **T** except for **r** has a parent node **u**.
  - Tree cannot be empty!
    - ⋆ Will always have at least the **root** node
- Definitions
  - **Child**
    If node u is the parent of node v, then v is a child of u.
  - **Siblings**:
    Two nodes which have the same parent.
  - **External Node**:
    A node which has no children.
    - Also known as a **leaf**.
  - **Internal Node**:
    A node which has at least one child.

Trees: Terminology and Basic Properties (continued)

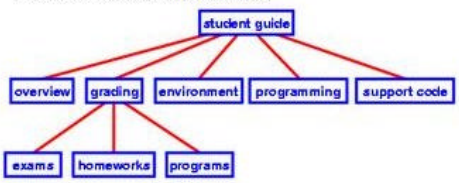
- Definitions (continued)
  - **Ancestor**:
    Either the node itself or an ancestor of the parent of the node.
  - **Descendant**:
    A node v is a descendant of a node u if u is an ancestor of v
  - **Descendant**:
    A node v is a descendant of a node u if u is an ancestor of v
  - **Subtree:**
    The subtree of tree T rooted at a node v is the tree consisting of all descendants of v in T (including v itself).
  - **Ordered Tree**:
    A linear ordering defined for the children of each node. We can identify the children as being the first, second, third etc.
  - Note the Recursive Definitions for **Ancestor**, **Descendant** and **Subtree**!

Trees: Examples

- organization structure of a corporation

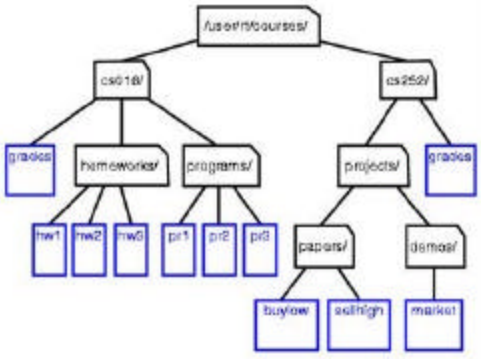


- table of contents of a book



Trees: Another Example

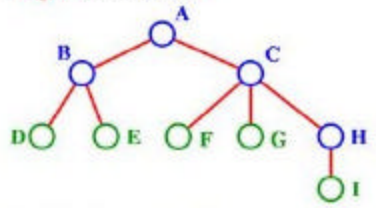
- Unix or DOS/Windows file system



- Internal nodes are associated with directories.
- External nodes are associated with regular files.
- In Unix, root of the tree is called ***root directory***.

Trees: Terminology

- *A* is the *root* node.
- *B* is the *parent* of D and E.
- *C* is the *sibling* of B
- *D* and *E* are the *children* of B
- *D, E, F, G, I* are *external nodes*, or *leaves*
- *A, B, C, H* are *internal nodes*
- The *depth* (*level*) of *E* is *2*
- The *height* of the tree is *3*
- The *degree* of node *B* is *2*



Property: (*# edges*) = (*#nodes*) – 1

Trees: ADT (1)

- Tree ADT stores elements at **positions**
  - Defined relative to neighboring **positions**.
- **Positions** in a tree are its nodes
  - Neighboring **positions** satisfy the parent-child relationships that define a valid tree.
- **Position** and **Node** are used interchangeably for trees!
- Methods
  - A **position** object for a tree supports the method:

    element()
    Return the object at this position
    Input: None, ***Output***: Object
  - Accessor methods

Trees: ADT (2)

root()
Return the root of the tree.
Input: None, ***Output***: Position

parent(v)
Return the parent of node v; An error occurs if v is the root.
Input: Position, ***Output***: Position

children(v)
Return the ***Iterator*** of the children of node v.
Input: Position, ***Output***: Iterator of Position

- Query Methods

isInternal(v)
Test whether node v is internal.
Input: Position, ***Output***: Boolean

Trees: ADT (3)

isExternall(v)
Test whether node v is external.
Input: Position, ***Output***: Boolean

isRoot(v)
Test whether node v is the root of the tree..
Input: Position, ***Output***: Boolean

- Generic Methods
  - Not necessarily related to a tree structure

size()
Return the number of nodes in the tree..
Input: None, ***Output***: Integer

elements()
Return an iterator of all elements stored in the nodes of the tree.
Input: None, ***Output***: Iterator of Objects

Trees: ADT (4)

positions()
Return the iterator of all nodes in the tree.
Input: None, ***Output***: Iterator of positions

swapElements(v, w)
Swap the elements stored at nodes v and w.
Input: Two positions, ***Output***: None

replaceElement(v, e)
Replace with e and return the element stored at node v.
Input: Position and an object, ***Output***: Object

Trees: Java Interface (1)

```
public interface InspectablePositionalContainer extends
    InspectableContainer {

    // accessor methods
    /** return the positions in the container */
    public PositionIterator positions();
}

public interface PositionalContainer extends
    InspectablePositionalContainer {

    // update methods
    /** swap the elements at two nodes */
    public void swapElements(Position v, Position w);

    /** replace with and return the element at a node */
    public Object replaceElement(Position v, Object e);
}
```


Trees: Java Interface (2)

```
public interface InspectableTree extends
     InspectablePositionalContainer {

    // accessor methods
    /** return the root of the tree */
    public Position root();

    /** return the parent of a node */
    public Position parent(Position v);
    /** return the children of a node */
    public PositionIterator children(Position v);

    // query methods
    /** test whether a node is internal */
    public boolean isInternal (Position v);

    /** test whether a node is external */
    public boolean isExternal(Position v);

    /** test whe her a node is the root of the tree */ public
    boolean isRoot(Position v);
}
```


Trees: Java Interface (3)

```
public interface Tree extends InspectableTree,
    PositionalContainer {}
```

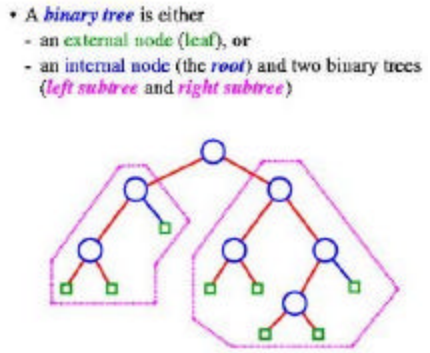
- Additional update methods may be added depending on the application.
  - Not included in the interface!
  - Tree interface is simply the combination of two other interfaces.

Trees: Binary Trees (1)

- Binary Tree
  - ◆ Ordered tree.
  - ◆ Each node has a maximum of two children.
- Definitions:
  - ◆ **Proper** Binary tree:
    Each node has either zero or two children.Every internal node has exactly two children.
  - ◆ **Left or Right Child**
    Each child of a node is labeled as either the left or right child. Left child comes before the right child.
  - ◆ **Left and Right Subtree**
    The subtree rooted at a left or right child of an internal node v.

Trees: Binary Trees (2)

- Recursive Definition:



Trees: Binary Tree ADT

- Accessor Methods:

  leftChildl(v)
  Return left child of node v; Error ocurrs if v is an external node.
  Input: Position, ***Output***: Position

  rightChildl(v)
  Return right child of node v; Error ocurrs if v is an external node.
  Input: Position, ***Output***: Position

  sibling(v)
  Return sibling of node v; Error ocurrs if v is the root.
  Input: Position, ***Output***: Position

- Specialized update methods are not defined here!
- May have additional error conditions if the trees are not proper!

Trees: Binary Tree Java Interface

```
public interface InspectableBinaryTree extends
    InspectableTree {

    // accessor methods
    /** return the left child of a node */
    public Position leftChild(Position v);

    /** return the right child of a node */  public Position
    rightChild(Position v);

     /** return the sibling of a node */  public Position
    sibling(Position v);}

public interface BinaryTree extends
    InspectableBinaryTree, PositionalContainer
```

Pseudo-Code (1)

- Description of an algorithm hat is more structured than usual prose but less formal than a programming language.
- Example: finding the maximum element of an array:

```
Algorithm arrayMax(A, n):
    Input: An array A storing n integers.
    Output: The maximum element in A.
  currentMax ← A[0]
  for i ← 1 to n –1 do
    if currentMax < A[i] then
      currentMax ← A[i]
  return currentMax
```

- More compact than Java code
  - Easier to read
- High level description permits high level analysis of a data structure or algorithm

Pseudo-Code (2)

- **What is Pseudo-Code?**
  - A mixture of natural language and high level programming concepts that describe the main ideas behind a generic implementation of a data structure or algorithm
  - **Expressions:** Use standard mathematical symbols to describe numeric and boolean expressions.
    - ⋆ Use ← for assignment ("=" in Java)
    - ⋆ Use = for equality relationships ("==" in Java)
  - Method Declarations:
    - ⋆ **Algorithm**(param1, param2 …)
  - Programming Constructs:
    - ⋆ Decision Structures **if … then … [else]**
    - ⋆ While Loops **while … do**
    - ⋆ For Loops **for … do**
    - ⋆ Array Indexing **A[I]**

Pseudo-Code (3)

- **What is Pseudo-Code?**
  - Methods:
    - ⋆ Calls **object.method(args)**
    - ⋆ Return **return** value
- When writing Pseudo Code, Keep in Mind:
  - Writing for a human reader, not a computer!
    - ⋆ Want high level ideas not low level implementation details!
    - ⋆ Don't forget important steps!
  - Skill which is improved with practice!

Array Based Implementations: Stack (1)

- Specify a maximum size **N** for the Stack, e.g. **N = 1000**
- Stack consists of an N element Array **S** and an integer variable t, the index of the top element in the Stack.
  - Array indices start with 0 so initialize t to –1.


```
Algorithm size():
  return t + 1

Algorithm isEmpty():
  return (t < 0)

Algorithm top():
  if isEmpty() then
    throw a StackEmptyException
  return S[t]
  …
```

Array Based Implementations: Stack (2)

```
Algorithm push(o):
  if size() = N then
    throw a StackFullException
  t ← t + 1
  S[t] ← o

Algorithm pop():
  if isEmpty() then
    throw a StackEmptyException
  e←S[t]
  S[t]←null
  t←t-1
  return e
```

Each of the above methods runs in O(1) time.
Very simple and efficient

Upper bound to the stack size may be:
Too small
May be too large and actually waste memory

Note the exceptions!

Array Based Implementations: Stack (3)

- **Source Code**

```
public class ArrayStack implements Stack {
  // Implementation of the stack interface
  // using an array.

  public static final int CAPACITY = 1024; // default
                    // capacity of the stack
  private int capacity; // maximum capacity of the
                    // stack.
  private Object S[ ]; // S holds the elements of
                    // the stack
  private int top = -1; // the top element of the
                    // stack.

  public ArrayStack( ) { // Initialize the stack
    this(CAPACITY);// with default capacity
  }

  public ArrayStack(int cap) { // Initialize the
                    // stack with given capacity
    capacity = cap;
    S = new Object[capacity];
  }
```


Array Based Implementations: Stack (4)

- **Source Code (continued)**

```
public int size( ) { //Return the current stack size
      return (top + 1);
}
public boolean isEmpty( ) {  // Return true iff
                             // the stack is empty
      return (top < 0);
}
public void push(Object obj)
      throws StackFullException{  // Push a new
                                  // element on the stack
      if (size() == capacity) {
        throw new StackFullException("Stack overflow.");
      }
      S[++top] = obj;
}
public Object top( )     // Return the top stack
                         // element
      throws StackEmptyException {
      if (isEmpty( )) {
        throw new StackEmptyException("Stack is
          empty.");
        }
      return S[top];
}
```


Array Based Implementations: Stack (5)

- **Source Code (continued)**

```
public Object pop() // Pop off the stack element
    throws StackEmptyException {
  Object elem;
  if (isEmpty( )) {
    throw new StackEmptyException("Stack is Empty.");
  elem = S[top];
  S[top--] = null; // Dereference S[top] and
                   // decrement top
  return elem;
  }
}
```

Array Based Implementations: Stack (6)

- Casting with a Generic Stack
  - Can store generic objects in the stack, each belonging to an arbitrary class.
  - Elements that are stored in it are viewed as instances of the Java Object class!
  - No trouble adding elements to the stack since every class in Java inherits from Object.
  - When removing an element from a Stack we get back a reference of type Object no matter what type of class the object may be.
  - Must **cast** the object to the specific class.

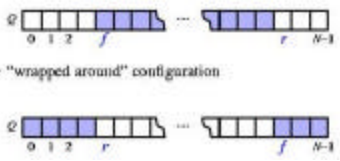
```
public static Integer[] reverse(Integer [] a){
  ArrayStack S = new ArrayStack(a.length);
  Integer[] b = new Integer(a.length);
  for(int i = 0; i < a.length; i++){
    S.push(a[I]);
  }
  for(int i = 0; i < a.length; i++){
    b[I] = (Integer) (S.pop());
  }
```


Array Based Implementations: Queue (1)

- Array **Q** to with a capacity **N**
- How do we keep track of the front and rear of the Q?
  - Can use same approach as the Stack – let Q[0] be the front of the queue and let the queue grow from there.
    - Not efficient! Need to move all the elements forward one array cell after a **dequeue** operation.
  - To avoid moving the objects:
    - Define two integer variables **f** and **r** with the following properties:
      f, index of the front element.
      r, index of the rear element.
  - Initially assign f = r = 0 to indicate queue is empty.
  - When we remove an element from front of the queue, increment f to index the next cell.
  - When we add an element, increment r to index the next available cell
  - Allows for O(1) time enqueue and dequeue

Array Based Implementations: Queue (2)

- Problem with this approach!
  - What if we enqueue and dequeue a single element N different times?
  - Then f = r = N!
  - If we tried to insert another element, we get an **ArrayIndexOutOfBoundsException**!
  - Still plenty of room in the array!
- Use a "Circular Array"
  - f and r indices wrap around the end of the queue.
  - Array goes from Q[0] – Q[N-1] and then immediately back to Q[0] again.



• "wrapped around" configuration

Array Based Implementations: Queue (3)

- Each time we increment f or r, compute the increment as:

  f = (f + 1) *mod* N
  r = (r + 1) *mod* N

- Now have O(1) time enqueue and dequeue
- What if we enqueue N objects without dequeueing any of them?
  - Then f = r, same condition when queue is empty and cannot tell the difference between a full and empty queue.
  - Solution?
    - Insists that Q can never hold more than N-1 elements.
- Computing Size of the Queue:

  (N – f + r) *mod* N

  - Gives correct size both in normal configuration (f <= r) and wrapped around configuration (r < f)

Array Based Implementations: Queue (4)

- Pseudo-Code:

```
Algorithm size()
    return (N – f + r) mod N

Algorithm isEmpty()
    return (f = r)

Algorithm front()
    if isEmpty then
        throw a QueueEmptyException
    return Q[f]

Algorithm dequeue()
    if isEmpty then
        throw a QueueEmptyException
    temp ← Q[f]
    Q[f] ← null
    f ← (f + 1) mod N
    return temp
```


Array Based Implementations: Queue (5)

- Pseudo-Code (continued):

```
Algorithm enqueue(o)
    if size() = N - 1 then
        throw a QueueFullException
    Q[r] ← o
    r ← (r + 1) mod N
```

- Disadvantages of Array based Implementations:
  - Set the capacity of the array
  - In a real application we may need more or less capacity than this.
- If we have a good estimate of the number of elements in the stack or array then array based implementations are simple and very efficient!

Stacks and Queue with Deques

Stack Methods	**Dequeue Method**
size()	size()
isEmpty()	isEmpty()
top()	last()
push(e)	insertLast(e)
pop()	removeLast()

Queue Method	**Dequeue Method**
size()	size()
isEmpty()	isEmpty()
front()	first()
enqueue()	insertLast()
dequeue()	removeFirst()

Bubble Sort Algorithm

- Bubble-Sort Algorithm
  - Sorts a sequence of elements in a sequence in non-decreasing order.
  - Performs a series of passes over the sequence
  - In each pass, elements are scanned in increasing rank, from rank 0 to the end of the sequence.
  - At each position in each pass, an element is compared with its neighbour
  - If in wrong order, elements are swapped
  - Total of n passes are performed
  - In first pass, when largest element is swapped, it will be swapped until it reaches the end of the sequence.
  - In the second pass, the seconds largest element is found etc.
  - Running Time:
    - ⋆ O(n^2)