

Assertions - Review: (2)			Loo
• Notation:			■ An
$\{P\} \to S \to \{Q\}$			eac loc
If <i>precondition</i> P is true before execution of statements S, then the <i>postcondition</i> Q will be true after execution.			
Ex	amples:		◆ .
1. 2.	$\{b=1\} \rightarrow b = b+1 \rightarrow \{b=2\}$ $\{true\} \rightarrow stack.push("y") \rightarrow$ $\{stack is non-empty\}$		
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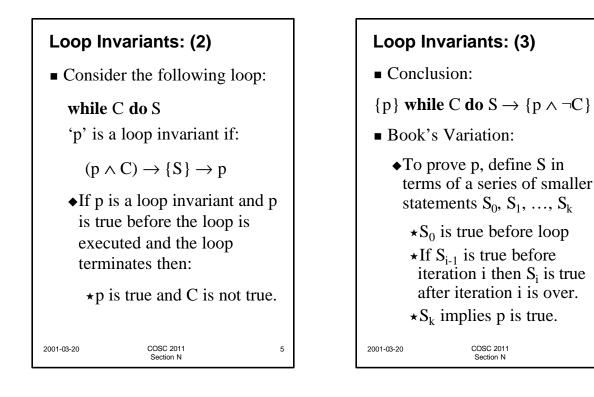
Loop Invariants: (1)

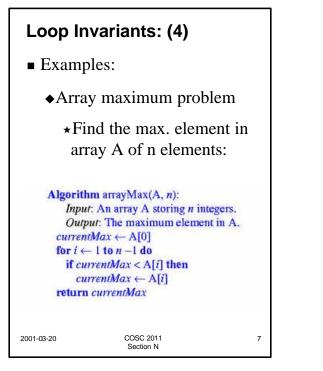
- An assertion that remains true each time the statements of a loop are executed
 - ◆Tells us something about the values of the loop variables while executing a loop.
 - Should be true at the beginning of each iteration, including the first!

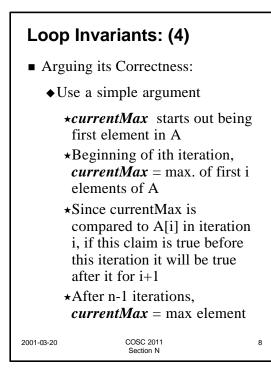
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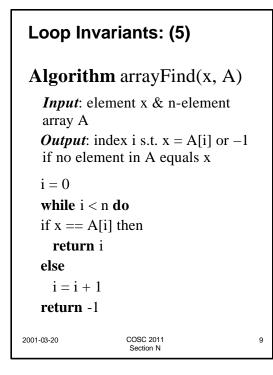
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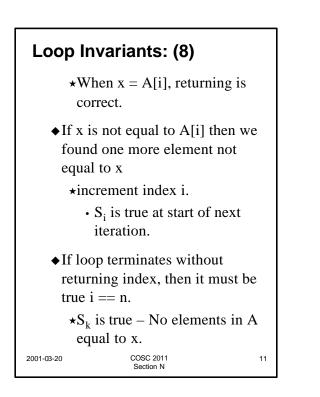


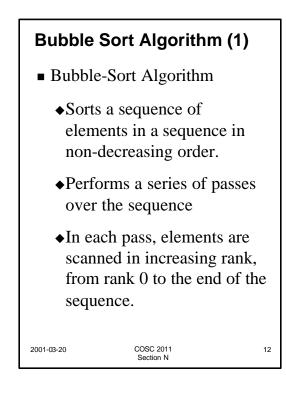


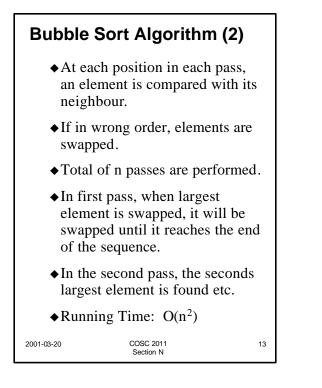
Loop Invariants: (7) Correctness (Book Method): Define series of statements S_i: *Claim*: At the beginning of each iteration i: S_i: x is not equal to any of the first i elements of A True at beginning of first iteration since no elements among the first 0 in A In iteration i, compare x to A[i] & return i if they are equal

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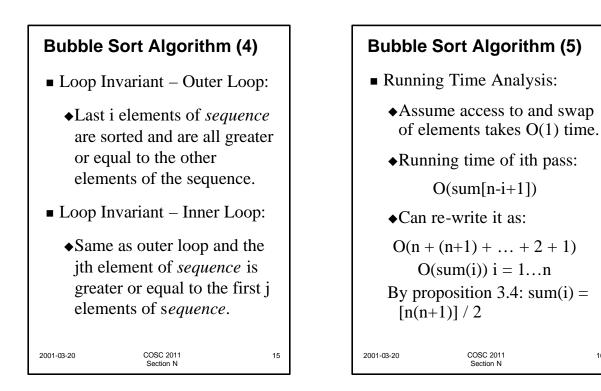
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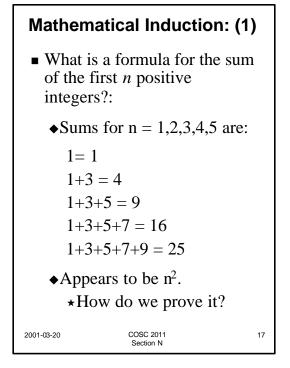


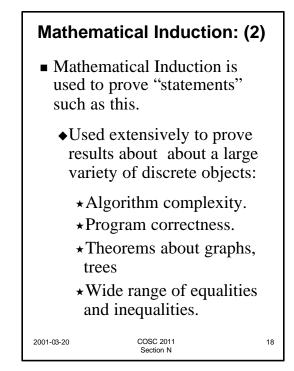


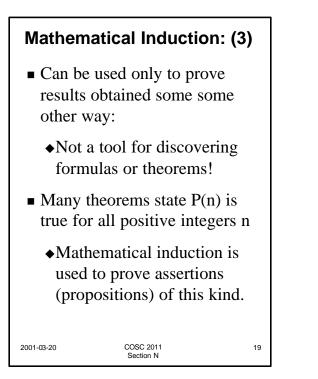


Bubble Sort Algorithm (3) Algorithm Bubblesort(sequence): *Input:* sequence of integers *sequence* **Postcondition**: sequence is sorted & contains the same integers as the original sequence *length* = length of *sequence* for i = 0 to length - 1 do for j = 0 to length - i - 2 do **if** *j*th element of *sequence* > (j+1)th element of sequence then swap *j*th and (j+1)th element of *sequence* 2001-03-20 COSC 2011 Section N 14





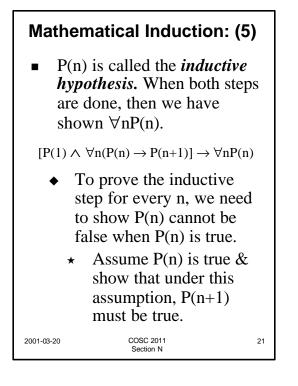




Mathematical Induction: (4) Used top prove statements of the form $\forall nP(n)$, for all positive integers.. A proof by mathematical induction that P(n) is true for all positive integers consists of two steps Basis Step: show P(1) (or 1. n =some other finite value) is true. Inductive Step: Show 2. $P(n) \rightarrow P(n+1)$ is true for every positive integer n. COSC 2011

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Mathematical Induction: (6)

- Remark: It is not assumed P(n) is true for all positive integers! Only shown that if it is assumed P(n) is true then P(n+1) is also true.
- When using induction, we show that P(1) is true. Then since P(1) implies P(2), P(2) must be true. Then P(3) is true because P(2) implies P(3). Continuing along these lines, P(k) is true for any positive integer k.

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Mathematical Induction: (7)		
■ Us	seful Illustration:	
٠	• Consider a line of people, person 1, person 2 etc. A secret is told to the first person and each person tells the secret to the next person in line.	
• Let P(n) be the statement that person n knows the secret.		
+	P(1) is true since it was told to first person.	
+	P(2) is true since person 1 tells person 2 and so on	
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Mathematical Induction: (9)

Another Illustration:

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- Infinite row of dominos, labeled 1,2,3,...,n & each domino is standing up.
- Let P(n) be the statement that domino n is knocked over.
- If the first domino is knocked over, P(1) is true.
- If whenever first domino is knocked over – P(1) is true, it knocks the (n+1)th domino over – P(n) → P(n+1) is true, then all dominos are knocked over!

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