COSC 2011 Section N

Thursday, March 29 2001

Overview

- Binary Tree traversals
  - Preorder, postorder, inorder
- Binary Tree Data Structures
  - Vector, Linked List
- General Tree Data Structures
- Converting General Trees to Binary Trees

Binary Tree Traversals: (1)

- ***Preorder*** Traversal :
  - Since a binary tree is also "regular tree", can use preorder traversal for general trees. However we can simplify it!

Algorithm binaryPreorder(T, *v*)
  perform visit action on node *v*
  if *v* is an internal node **then**
    binaryPreorder(T, T.leftChild(*v*))
    binaryPreorder(T, T.rightChild(*v*))

Binary Tree Traversals: (2)

- ***Postorder*** Traversal :
  - Can also simplify the postorder traversal for binary trees.

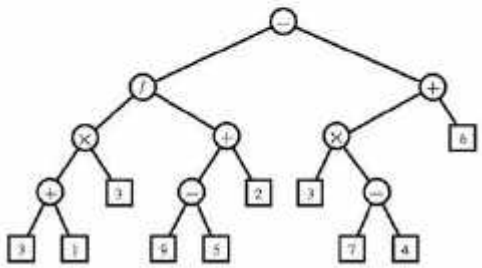
Algorithm binaryPostorder(T, *v*)
  if *v* is an internal node **then**
    binaryPostorder(T, T.leftChild(*v*))
    binaryPostorder(T,T.rightChild(*v*))
  perform visit action on node *v*

  - Can be used to solve the expression evaluation problem.

Binary Tree Traversals: (3)

- specialization of a postorder traversal

Algorithm evaluateExpression(v)
  if v is an external node
    return the variable stored at v
  else
    let o be the operator stored at v
    x ← evaluateExpression(leftChild(v))
    y ← evaluateExpression(rightChild(v))
    return x o y



Binary Tree Traversals: (4)

- ***Inorder*** Traversal :
  - Visit a node between the recursive traversals of its left and right subtrees.

Algorithm inorder(T, *v*)
  if *v* is an internal node **then**
    inorder(T, T.leftChild(*v*))
  perform "visit" for node *v*
  if *v* is an internal node **then**
    inorder(T, T.rightChild(*v*))

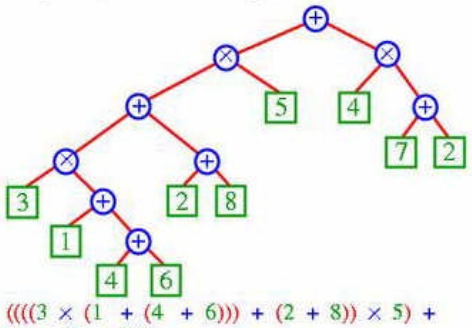
Binary Tree Traversals: (5)

- Visit the nodes of T "from left to right".
- Visits *v* after all nodes in its left subtree and before the nodes of its right subtree.
- Many Applications:
  - ⋆ Inorder traversal of a ***binary search tree*** visits the elements in a non-decreasing order.
  - ⋆ Tree Drawing

Binary Tree Traversals: (6)

- Example: Printing an Arithmetic Expression

- specialization of an inorder traversal
- print "(" before traversing the left subtree
- print ")" after traversing the right subtree



((((3 × (1 + (4 + 6))) + (2 + 8)) × 5) + (4 × (7 + 2)))

Binary Search Tree: (1)

- Definition:
  - Each internal node *v* stores an element *e* such that:
    - ⋆ Elements stored in the left subtree of *v* are less than or equal to *e*.
    - ⋆ Elements stored in the right subtree of *v* are greater than or equal to *e*.

Binary Tree Data Structures: (1)

- Vector Based Implementation:
  - ***Level Ordering***: For every node v of T, let *p(v)* be the integer defined as follows:
    - If *v* is the root, *p(v)* = 1.
    - If *v* is left child of node *u*, *p(v)* = 2*p(u)*.
    - If *v* is right child of node *u*, *p(v)* = 2*p(u)* + 1
    - Numbers the nodes of each level of T in increasing order from left to right (but may skip some nodes!)

Binary Tree Data Structures: (2)

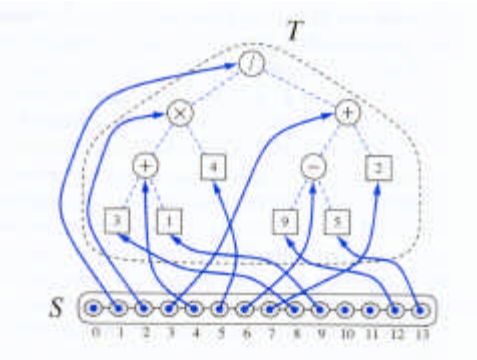
- Representation of binary tree T using a vector Such that node *v* of T is associated with element of S at rank *p(v)*.
- Simple and efficient implementation.
  - Perform the methods root, parent, leftChild, rightChild, sibling, isInternal, isExternal and isRoot using simple math on the numbers *p(v)*.

Binary Tree Data Structures: (4)

- Let n be number of nodes of T, p_M max. value of *p(v)* over all nodes of T.
  - Vector size N = p_M + 1
  - No element at rank 0!
- Vector method is fast and easy representation but can be very space inefficient if the height of the tree is large!

Binary Tree Data Structures: (5)

- Representation of a Binary tree T with a Vector S.



Binary Tree Data Structures: (6)

- Running Times of the Methods of a Binary Tree Implemented with a Vector:

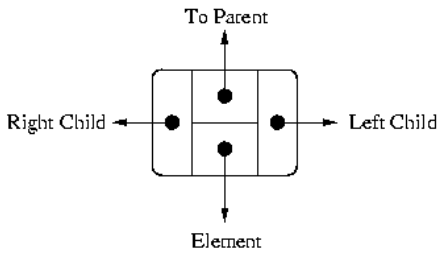
Operation	Time
positions, elements	O(n)
swapElements, replaceElements	O(1)
root, parent, children	O(1)
leftChild, rightChild, sibling	O(1)
isInternal, isExternal, isRoot	O(1)

Binary Tree Data Structures: (7)

- Linked Structure for Binary Trees:
  - Represent each node *v* of tree T by an object with reference to:
    - Element stored at *v*.
    - Position objects associated with the children and parent of *v*.

Binary Tree Data Structures: (8)

- Node Used in the Binary tree Linked List Implementation:



Node for a Binary Tree Linked List

Binary Tree Data Structures: (9)

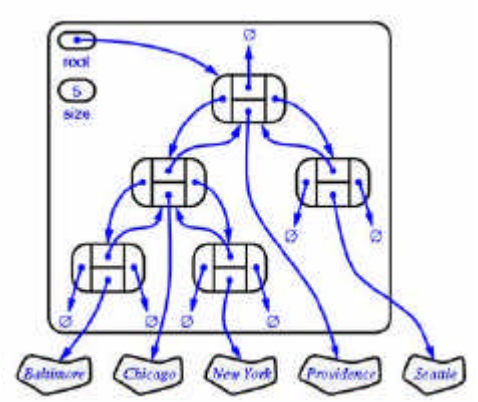
- If *v* is the root of T, reference to parent is null.
- If *v* is an external node of T, references to children are null.
  - To save space, when external nodes are empty, can have references to external nodes be null.
  - Can use a special object, *NULL_NODE* & every external node reference is instead to this object.

Binary Tree Data Structures: (10)

- Using the *NULL_NODE* we have to be prepared to throw an exception if the parent method is passed such an object as an argument.

Binary Tree Data Structures: (11)

- Example of a Linked Data Structure for a Binary Tree:

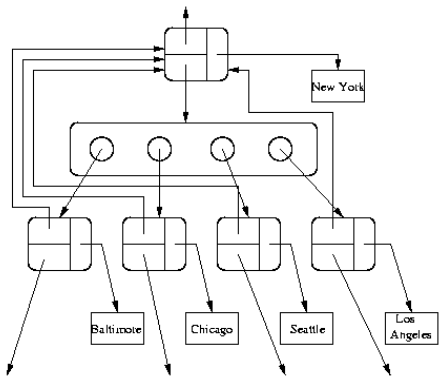


General Tree Data Structures: (1)

- Linked Structure for General Trees:
  - Can extend the linked structure for binary trees to represent general trees.
  - No limit to the number of children a node can have, use a container (e.g. **list, vector**) to store the children of node *v* instead of using instance variables.

General Tree Data Structures: (2)

- Linked Structure for General Trees:



Linked Structure for a General Tree

General Tree Data Structures: (3)

- Can implement method children(*v*) by simply calling elements() method of the container.

Operation	Time
Size, isEmpty	O(1)
Positions, elements	O(n)
swapElements, replaceElements	O(1)
Root, parent	O(1)
isInternal, isExternal, isRoot	O(1)
Children(*v*)	$O(c_v)$

Converting a General Tree to a Binary Tree: (1)

- Representing General Trees with Binary Trees. Transform T into Binary Tree T' as follows:
  - For each node *u* of T, there is an internal node *u'* of T' associated with *u.*
  - If *u* is an external node of T and doesn't have a sibling immediately following it, then the children *u'* of T' are external nodes.

Converting a General Tree to a Binary Tree (2)

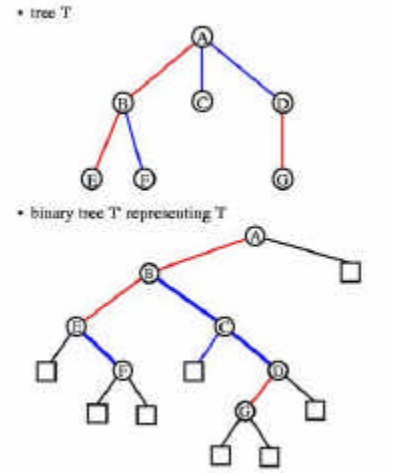
- If *u* is an internal node of T and *v* is the first child of *u* in T, then *v'* is the left child of *u'* in T.
- If node *v* has a sibling *w* immediately following it, then *w'* is the right child of *v'* in T'.
- External nodes of T' are not associated with nodes T and serve only as placeholders.

Converting a General Tree to a Binary Tree: (3)

- Can be seen as a conversion of T into T' that takes each set of siblings $\{v_1, v_2, \ldots, v_k\}$ in T with parent *v* and replaces it with a chain of right children rooted at v_1, which then becomes the left child of *v.*

Converting a General Tree to a Binary Tree: (4)

- Example of the Conversion from a General Tree to a Binary Tree:



Priority Queue: (1)

- What is a Priority Queue?
  - ◆An Abstract Data storing a collection of prioritized elements.
  - ◆Supports arbitrary element insertion but supports removal of elements in order of priority.
    - ⋆The element with the highest priority can be removed at any time.

Priority Queue: (2)

- ◆A priority queue stores elements in order of priority only!
  - ⋆No notion of *position* as with some other ADTs (sequences, lists etc.)
- Priority Queue ADT:
  - ◆Each element in the priority queue has a corresponding "***key***" Object.
    - ⋆Key Object represents the elements priority.

Priority Queue: (3)

- "***Key***" Object - Definition:
  - ◆An Object assigned to some element which can be used to *rank, identify* or *weight* the element.
  - ◆Assigned to the element by the user or the application.
  - ◆Maybe changed by the application if needed.

Priority Queue: (4)

- Does not need to be a single numerical value.
  - ⋆Can sometimes be more complex and cannot be quantified by a single number.

Priority Queue: (4)

- In a priority queue, the key is used to assign a priority to each element:

- A Priority Queue ranks its elements by ***key*** with a ***total order*** relation

- Keys:
  - Every element has its own key
  - Keys are not necessarily unique

- Total Order Relation
  - Denoted by $\leq$
  - **Reflexive:** $k \leq k$
  - **Antisymetric:** if $k_1 \leq k_2$ and $k_2 \leq k_1$, then $k_1 \leq k_2$
  - **Transitive:** if $k_1 \leq k_2$ and $k_2 \leq k_3$, then $k_1 \leq k_3$

Priority Queue: (5)

Sorting with a Priority Queue

- A Priority Queue P can be used for sorting a sequence S by:
  - inserting the elements of S into P with a series of insertItem(e, e) operations
  - removing the elements from P in increasing order and putting them back into S with a series of removeMin() operations

Algorithm PriorityQueueSort(S, P):
Input: A sequence S storing n elements, on which a total order relation is defined, and a Priority Queue P that compares keys with the same relation
Output: The Sequence S sorted by the total order relation

while !S.isEmpty() **do**
  $e \leftarrow S$.removeFirst()
  P.insertItem(e, e)
while P is not empty **do**
  $e \leftarrow P$.removeMin()
  S.insertLast(e)

Priority Queue: (6)

The Priority Queue ADT

- A prioriy queue P supports the following methods:

- size():
  Return the number of elements in P
- isEmpty():
  Test whether P is empty
- insertItem(k,e):
  Insert a new element e with key k into P
- minElement():
  Return (but don't remove) an element of P with smallest key; an error occurs if P is empty.
- minKey():
  Return the smallest key in P; an error occurs if P is empty
- removeMin():
  Remove from P and return an element with the smallest key; an error condidtion occurs if P is empty.