COSC 2011 Section N

Tuesday, April 3 2001

Overview

- Priority Queues
  - Quick Review of definitions
  - Quick Review of ADT
  - Compositions & Comparators
  - Implementation
    - Sequence
- Mid-term Test Questions

Priority Queues: Compositions (1)

- Composition Objects:
  - A single object *e* that is a composition of two (or more) other objects.
  - Each element in the priority queue is essentially a pair:
    - Each element has a *key*
  - Can create a composition of each key and element:

Priority Queues: Compositions (2)

- Implementing the Composition Concept:
  - Define the Item class:

```
public class Item{
  private Object key, elem;
  public Item(Object k, Object e){
    key = k;
    elem = e;
  }
  public Object key(){return key;}
  public Object element(){return element;}
  public void setKey(Object k){key = k;}
  public void setElem(Object e){elem = e;}
}
```


Priority Queues: Comparators (1)

- How do we Compare keys?
  - Keys are Objects so they may be different types!
  - Use different priority queue for each key type and each possible way of comparing keys of such types.
    - Different priority queue for integer keys, strings…
    - Not very general.
    - Too much similar code!

Priority Queues: Comparators (2)

- Alternative Strategy:
  - Requires keys to be able to compare themselves to one another.
  - Have a general priority queue class that stores instances of a key class that implements a ***Comparable*** interface.
    - Encapsulates all the usual comparison methods.

Priority Queues: Comparators (3)

- Problem with the Comparable:
  - May be cases where we are asking "too much" of the keys.
    - Keys may not know how to be compared!
    - $4 \leq 11$ using integer keys.
    - $11 \leq 4$ using String keys.
  - Do not rely on keys to provide comparison rules!

Priority Queues: Comparators (4)

- Use ***comparator*** objects:
  - External to the keys
  - Supply comparison rules.
  - Given to the priority queue during construction
  - Can be changed if necessary.
- When the priority queue needs to compare keys, it uses the ***comparator*** object.

Priority Queues: Comparators (5)

- Comparator ADT Methods:

- The comparator ADT includes:
  - isLessThan(a, b)
  - isLessThanOrEqualTo(a,b)
  - isEqualTo(a, b)
  - isGreaterThan(a,b)
  - isGreaterThanOrEqualTo(a,b)
  - isComparable(a)

Priority Queues: Sequence Implementation (6)

Implementation with an Unsorted Sequence

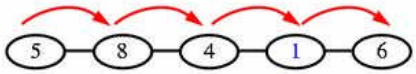
- Let's try to implement a priority queue with an unsorted sequence S.
- The elements of S are a composition of two elements, k, the key, and e, the element.
- We can implement insertItem() by using insertLast() on the sequence. This takes $O(1)$ time.

- However, because we always insert at the end, irrespectively of the key value, our sequence is not ordered.

Priority Queues: Sequence Implementation (7)

Implementation with an Unsorted Sequence (contd.)

- Thus, for methods such as minElement(), minKey(), and removeMin(), we need to ***look at all the elements*** of S. The worst case time complexity for these methods is $O(n)$.



- Performance summary

insertItem	$O(1)$
minKey, minElement	$O(n)$
removeMin	$O(n)$

Priority Queues: Sequence Implementation (8)

Implementation with a Sorted Sequence

- Another implementation uses a sequence S, sorted by increasing keys
- minElement(), minKey(), and removeMin() take $O(1)$ time

- However, to implement insertItem(), we must now scan through the entire sequence ***in the worst case***. Thus, insertItem() runs in $O(n)$ time

- Performance summary

insertItem	$O(n)$
minKey, minElement	$O(1)$
removeMin	$O(1)$

Priority Queues: Sorting (1)

Selection Sort

- Selection Sort is a variation of PriorityQueueSort that uses an ***unsorted sequence*** to implement the priority queue P.
- Phase 1, the insertion of an item into P takes $O(1)$ time
- Phase 2, removing an item from P takes time proportional to the current number of elements in P

		Sequence S	Priority Queue P
Input		(7, 4, 8, 2, 5, 3, 9)	()
Phase 1:			
	(a)	(4, 8, 2, 5, 3, 9)	(7)
	(b)	(8, 2, 5, 3, 9)	(7, 4)
	...	...	...
	(g)	()	(7, 4, 8, 2, 5, 3, 9)
Phase 2:			
	(a)	(2)	(7, 4, 8, 5, 3, 9)
	(b)	(2, 3)	(7, 4, 8, 5, 9)
	(c)	(2, 3, 4)	(7, 8, 5, 9)
	(d)	(2, 3, 4, 5)	(7, 8, 9)
	(e)	(2, 3, 4, 5, 7)	(8, 9)
	(f)	(2, 3, 4, 5, 7, 8)	(9)
	(g)	(2, 3, 4, 5, 7, 8, 9)	()

Priority Queues: Sorting (2)

Selection Sort (cont.)

- As you can tell, a bottleneck occurs in Phase 2. The first removeMinElement operation take $O(n)$, the second $O(n-1)$, etc. until the last removal takes only $O(1)$ time.
- The total time needed for phase 2 is:

$$O(n + (n-1) + \ldots + 2 + 1) \equiv O\left(\sum_{i=1}^{n} i\right)$$

- By a well-known fact:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

- The total time complexity of phase 2 is then $O(n^2)$. Thus, the time complexity of the algorithm is $O(n^2)$.

Priority Queues: Sorting (3)

Insertion Sort

- Insertion sort is the sort that results when we perform a PriorityQueueSort implementing the priority queue with a ***sorted sequence***.

	Sequence S	Priority Queue P
Input	(7, 4, 8, 2, 5, 3, 9)	()
Phase 1:		
(a)	(4, 8, 2, 5, 3, 9)	(7)
(b)	(8, 2, 5, 3, 9)	(4, 7)
(c)	(2, 5, 3, 9)	(4, 7, 8)
(d)	(5, 3, 9)	(2, 4, 7, 8)
(e)	(3, 9)	(2, 4, 5, 7, 8)
(f)	(9)	(2, 3, 4, 5, 7, 8)
(g)	()	(2, 3, 4, 5, 7, 8, 9)
Phase 2:		
(a)	(2)	(3, 4, 5, 7, 8, 9)
(b)	(2, 3)	(4, 5, 7, 8, 9)
...	...	...
(g)	(2, 3, 4, 5, 7, 8, 9)	()

Priority Queues: Sorting (4)

Insertion Sort(cont.)

- We improve phase 2 to $O(n)$.
- However, phase 1 now becomes the bottleneck for the running time. The first insertItem takes $O(1)$ time, the second one $O(2)$, until the last opertation takes $O(n)$ time, for a total of $O(n^2)$ time
- Selection-sort and insertion-sort both take $O(n^2)$ time
- Selection-sort will ***always*** executs a number of operations proportional to n^2, no matter what is the input sequence.
- The running time of insertion sort varies depending on the input sequence.
- Neither is a good sorting method, except for small sequences
- We have yet to see the ultimate priority queue....

Priority Queues: Sorting (5)

- By now, you've seen a little bit of sorting, so let us tell you a little more about it.
- Sorting is essential because efficient ***searching*** in a database can be performed only if the records are sorted
- It is estimated that about 20% of all the computing time worldwide is devoted to sorting
- We shall see that there is a trade-off between the "simplicity" and efficiency of sorting algorithms:
- The elementary sorting algorithms you've just seen, though easy to understand and implement, take $O(n^2)$ time (unusable for large values of n)
- more sophisticated algorithms take $O(n\log n)$ time
- Comparison of Keys: *do we base comparison upon the entire key or upon parts of the key?*
- Space Efficiency: ***in-place*** *sorting vs. use of auxiliary structures*
- Stability: ***a stable*** *sorting algorithm preserves the initial relative order of equal keys*