# COSC 2011 Section N

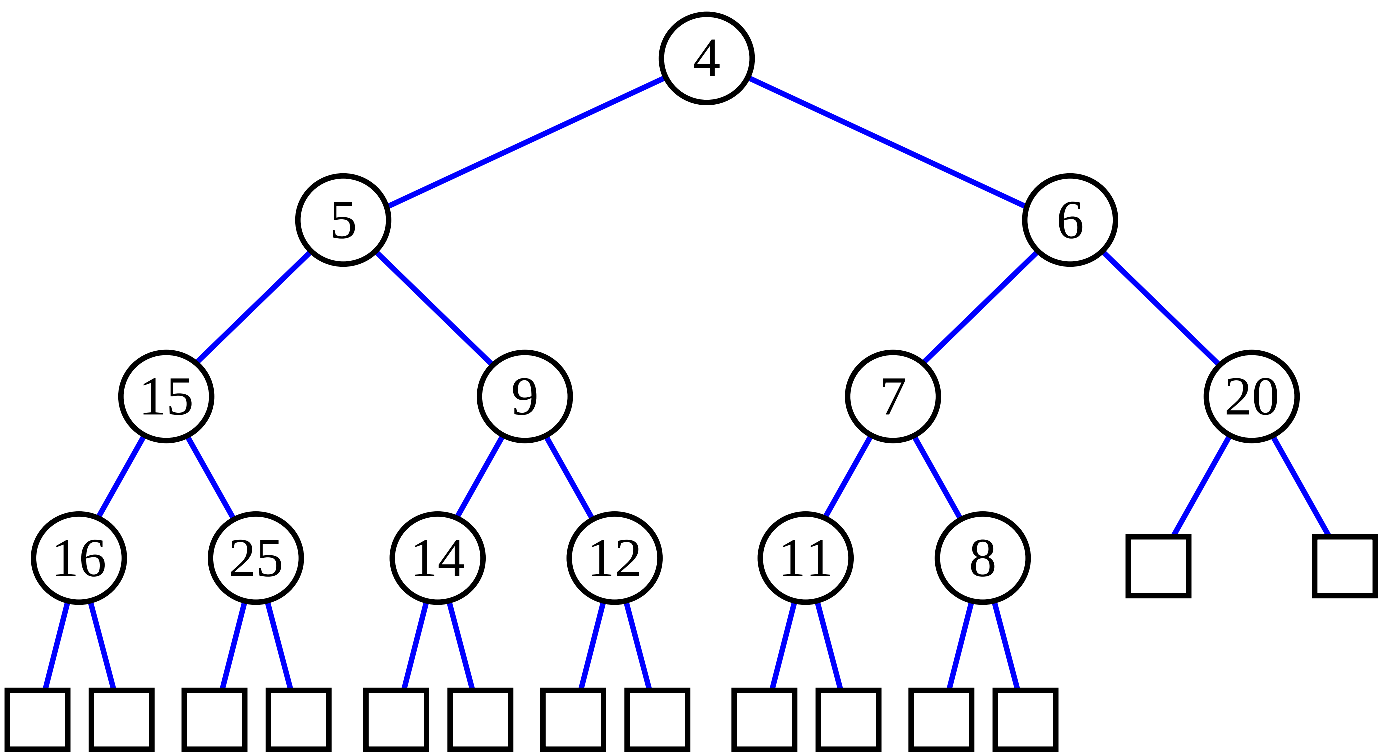
Tuesday, April 10 2001

## Overview

- Heaps
  - Definitions
  - Properties
  - Insertion / Deletion
  - Implementation
    - Vector
  - Heap Sort
- Dictionaries

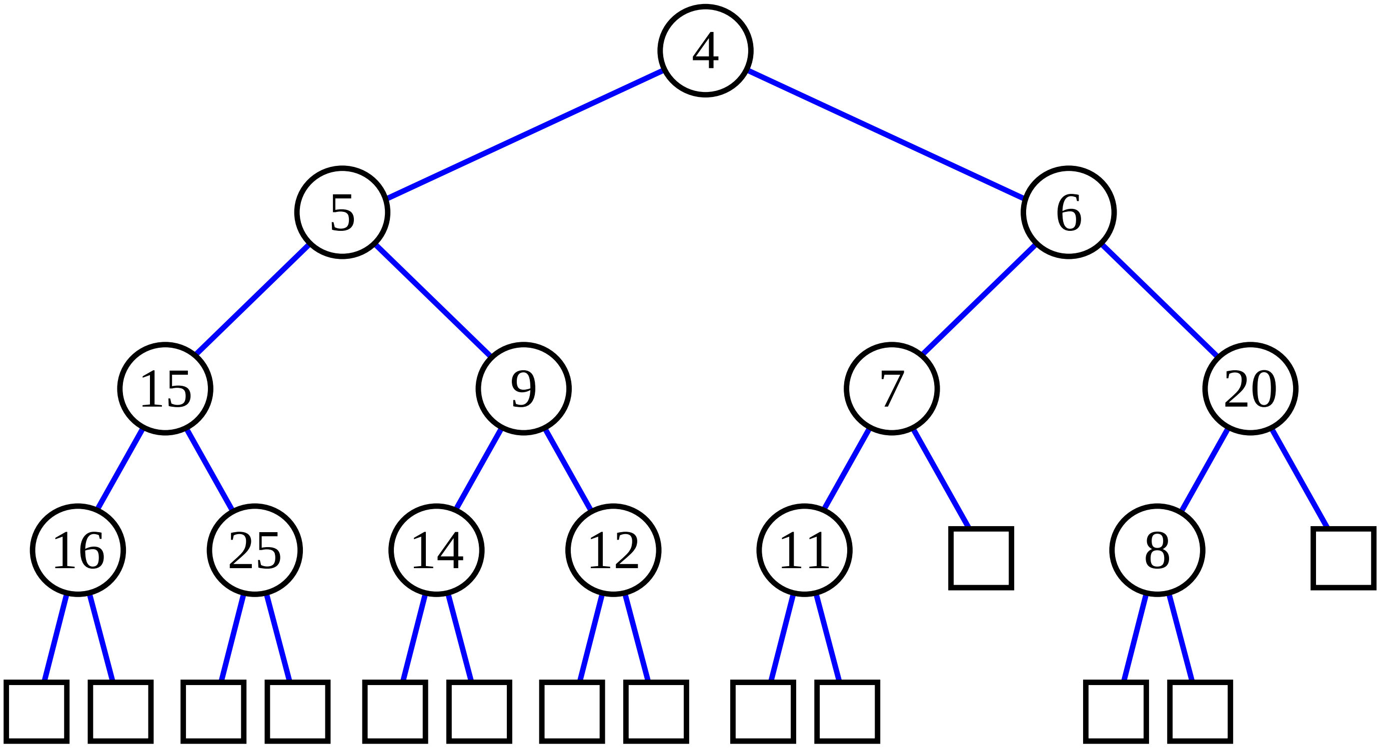
# Heaps

- A ***heap*** is a binary tree *T* that stores a collection of keys (or key-element pairs) at its internal nodes and that satisfies two additional properties:
  - ***Order Property:*** key(parent) ≤ key(child)
  - ***Structural Property***: all levels are full, except the last one, which is left-filled (***complete binary tree***)

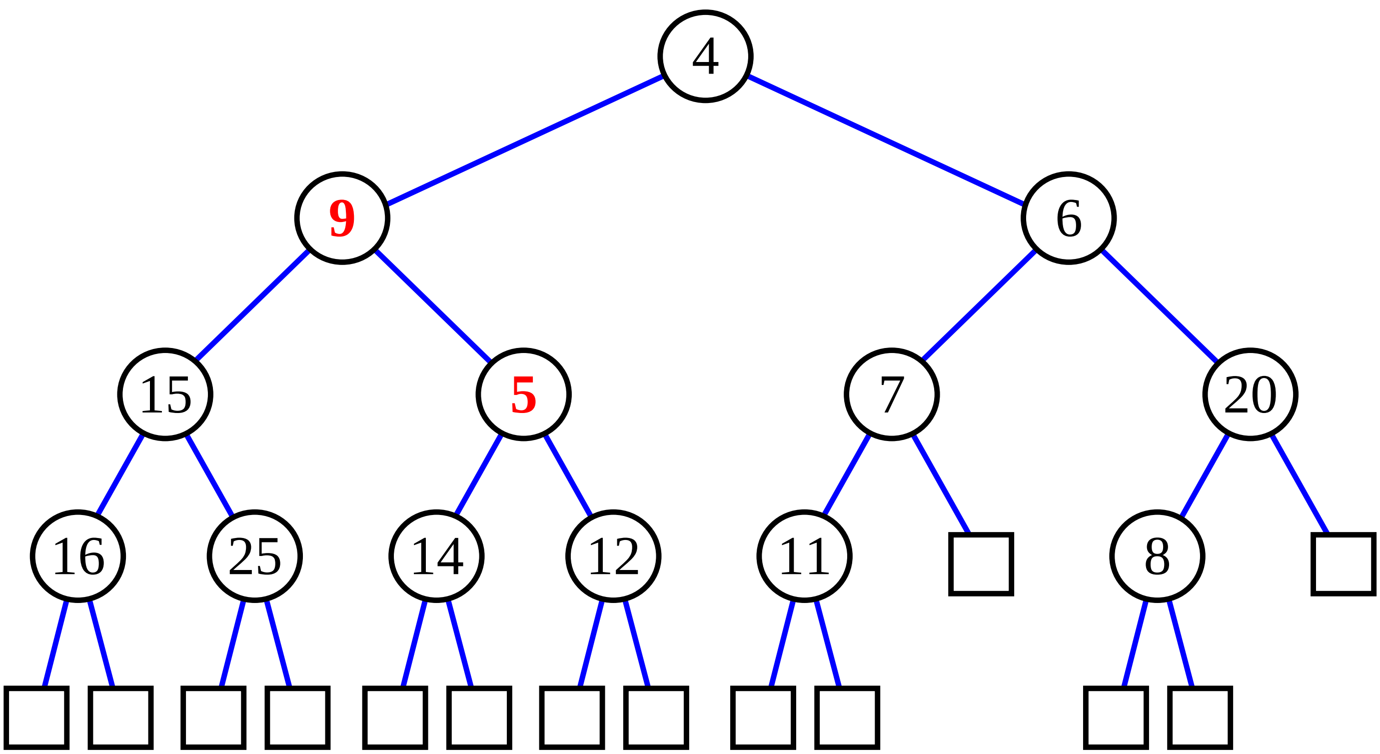


# Not Heaps

- bottom level is not left-filled



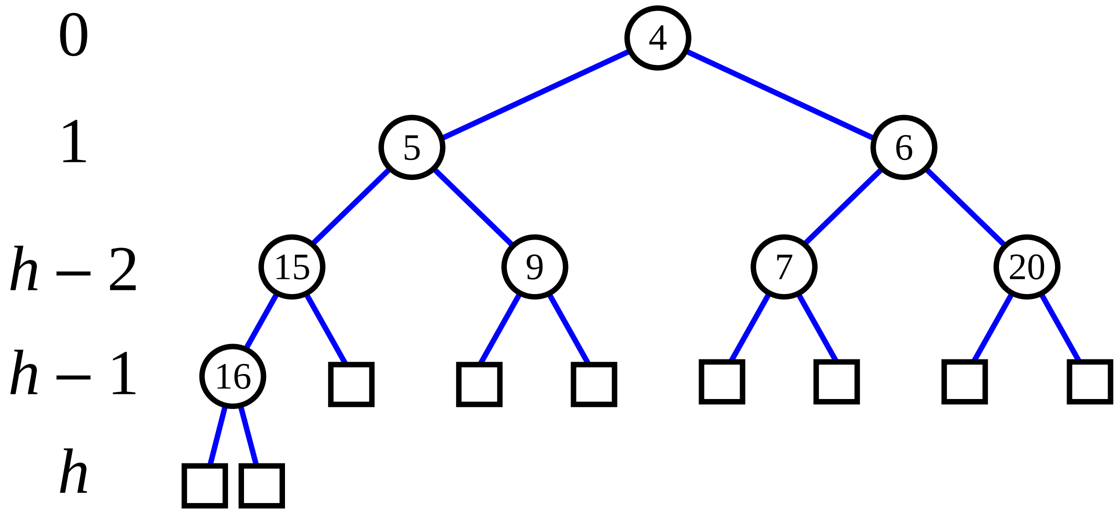

- key(parent)> key(child)



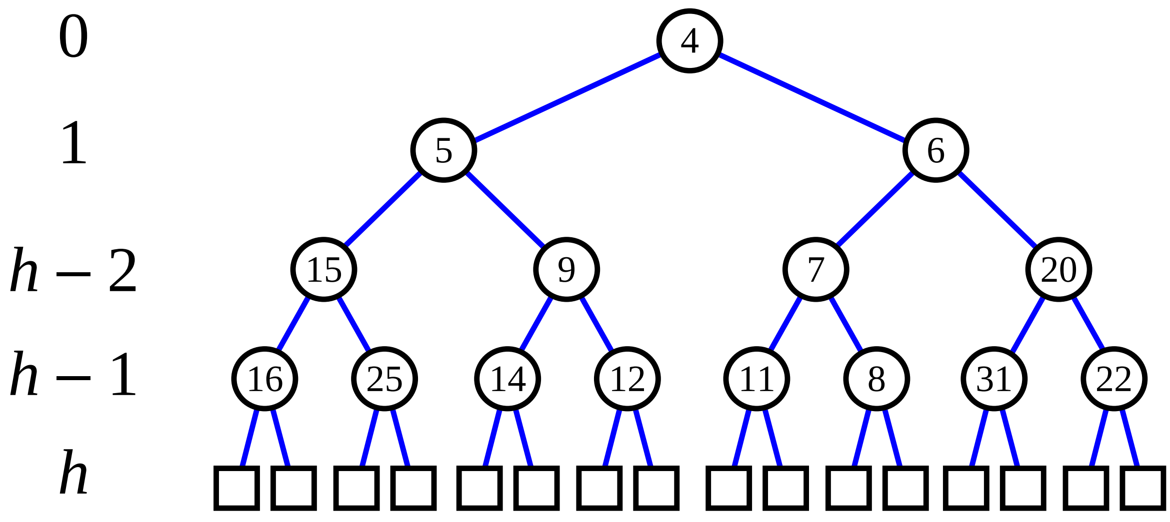
# Height of a Heap

A heap $T$ storing $n$ keys has height $h = \lceil \log(n + 1) \rceil$, which is $O(\log n)$

- $n \geq 1 + 2 + 4 + \ldots + 2^{h-2} + 1 = 2^{h-1} - 1 + 1 = 2^{h-1}$




- $n \leq 1 + 2 + 4 + \ldots + 2^{h-1} = 2^h - 1$

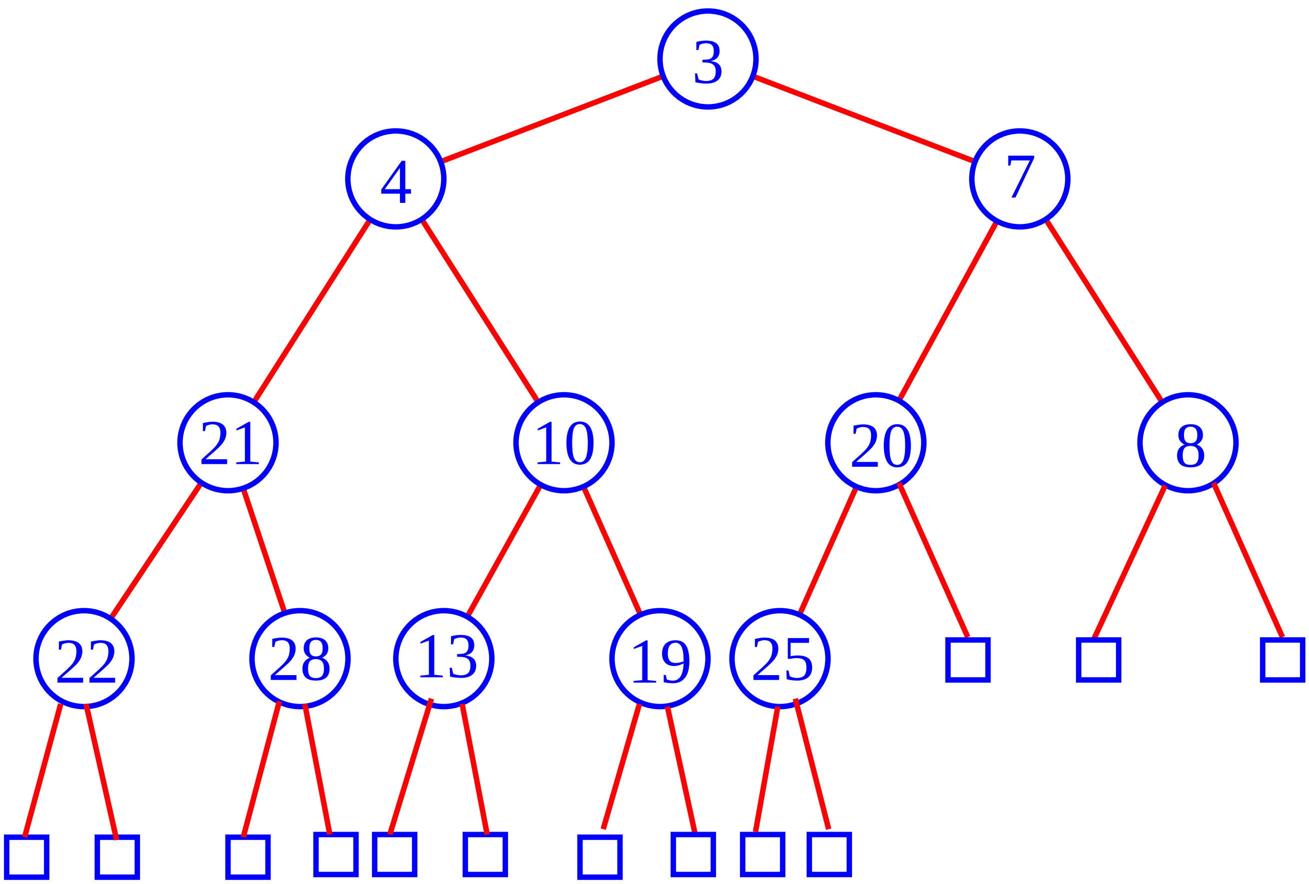



- Therefore $2^{h-1} \leq n \leq 2^h - 1$
- Taking logs, we get $\log(n + 1) \leq h \leq \log n + 1$
- Which implies $\boldsymbol{h = \lceil \log(n+1) \rceil}$

# Heap Insertion

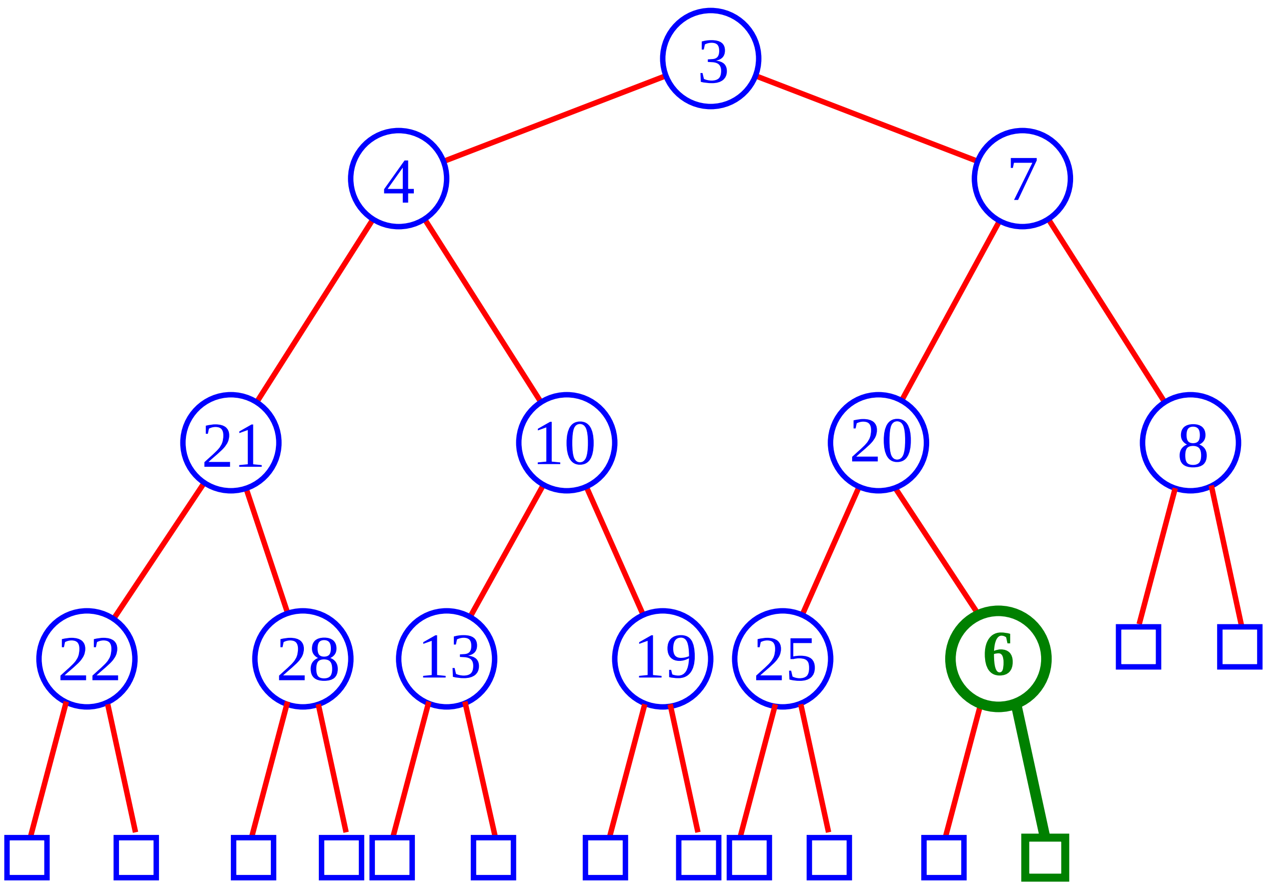
So here we go ...

The key to insert is **6**



# Heap Insertion

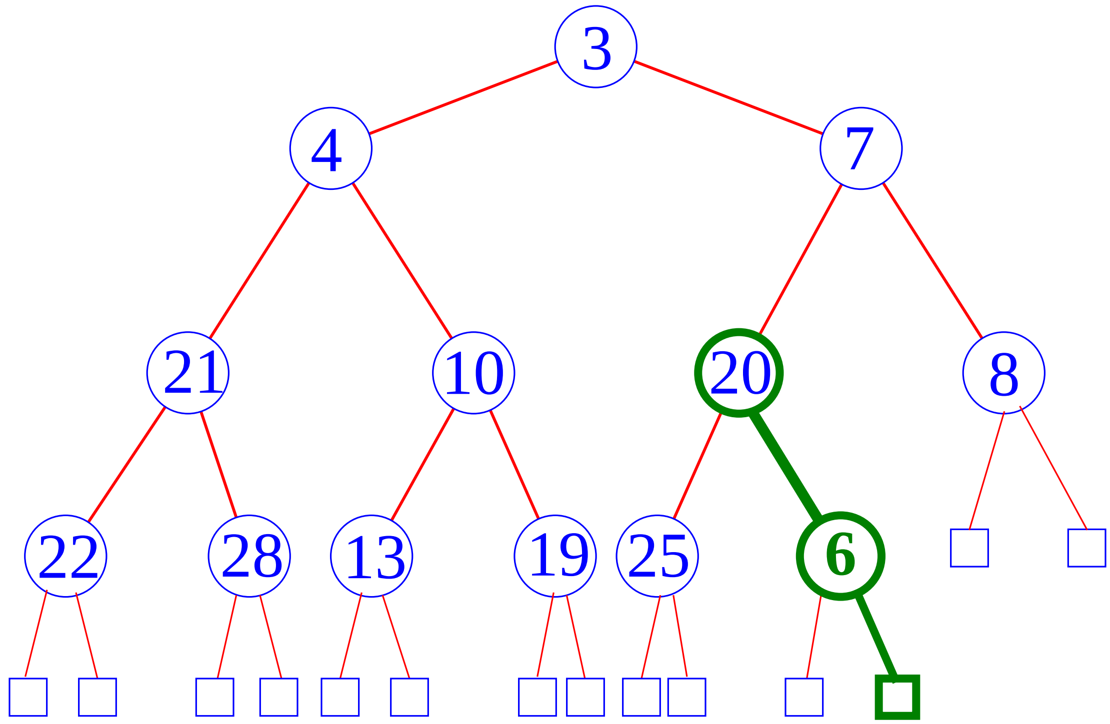
Add the key in the ***next available position*** in the heap.

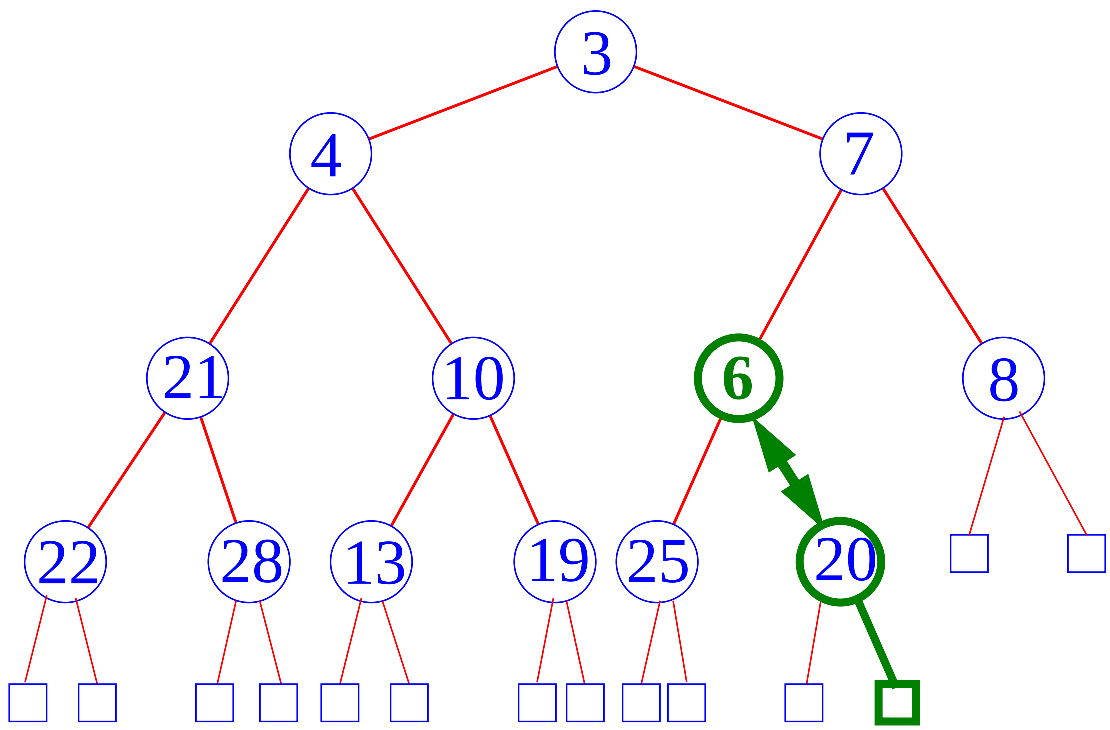



Now begin ***Upheap***.

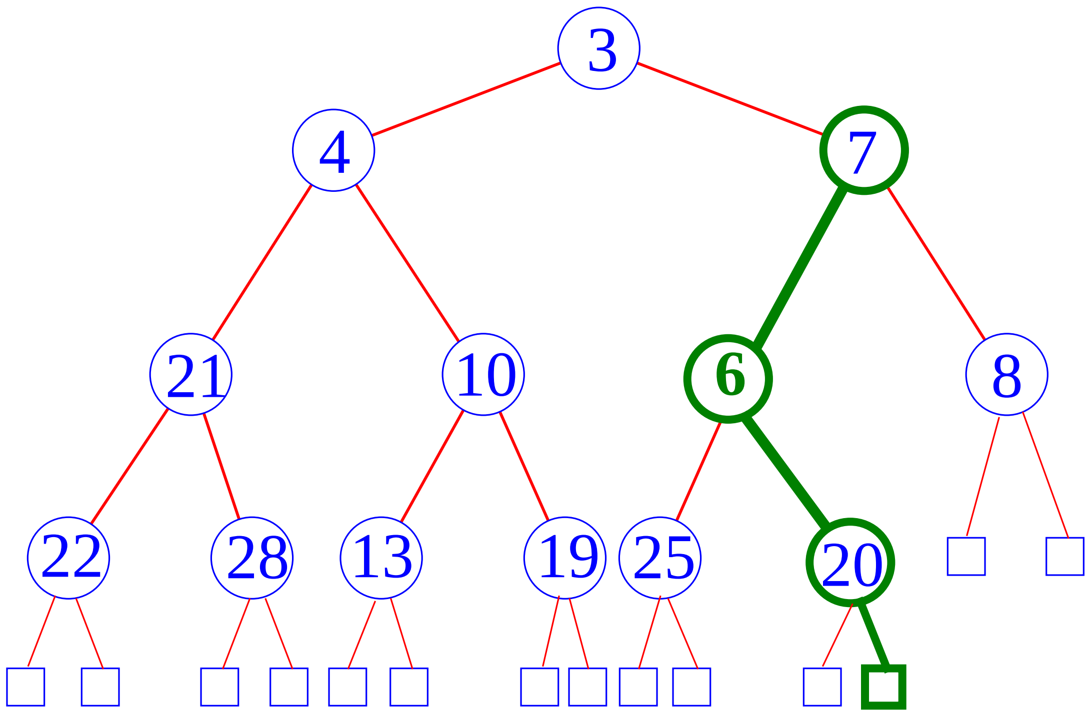
# Upheap

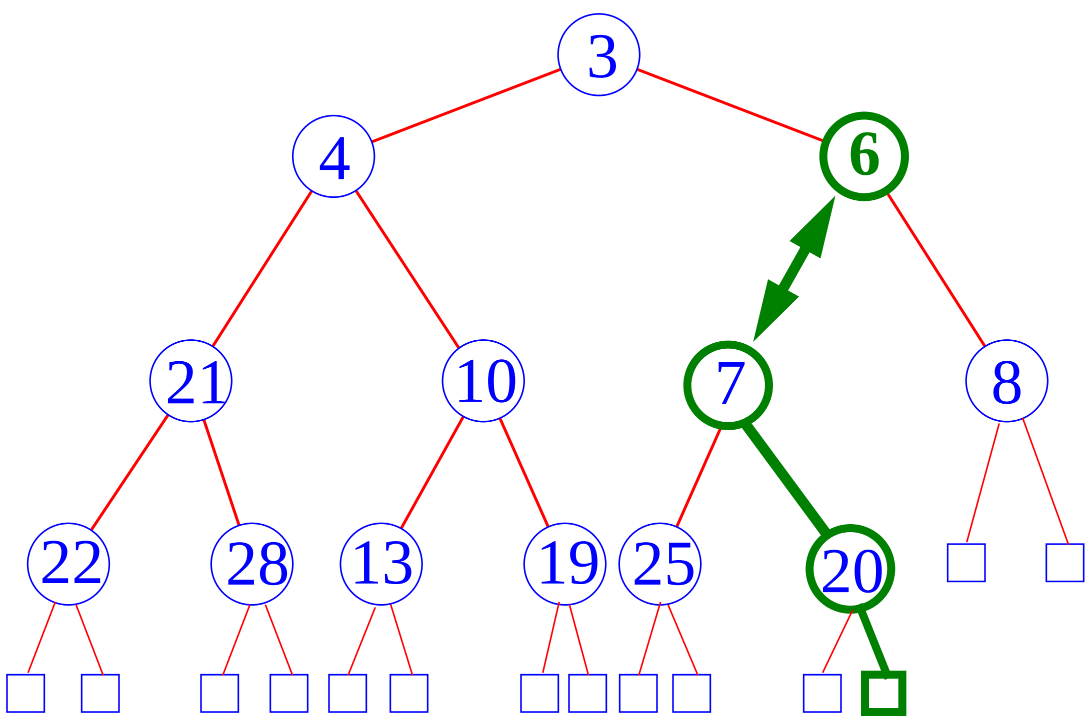
- ***Swap parent-child keys out of order***



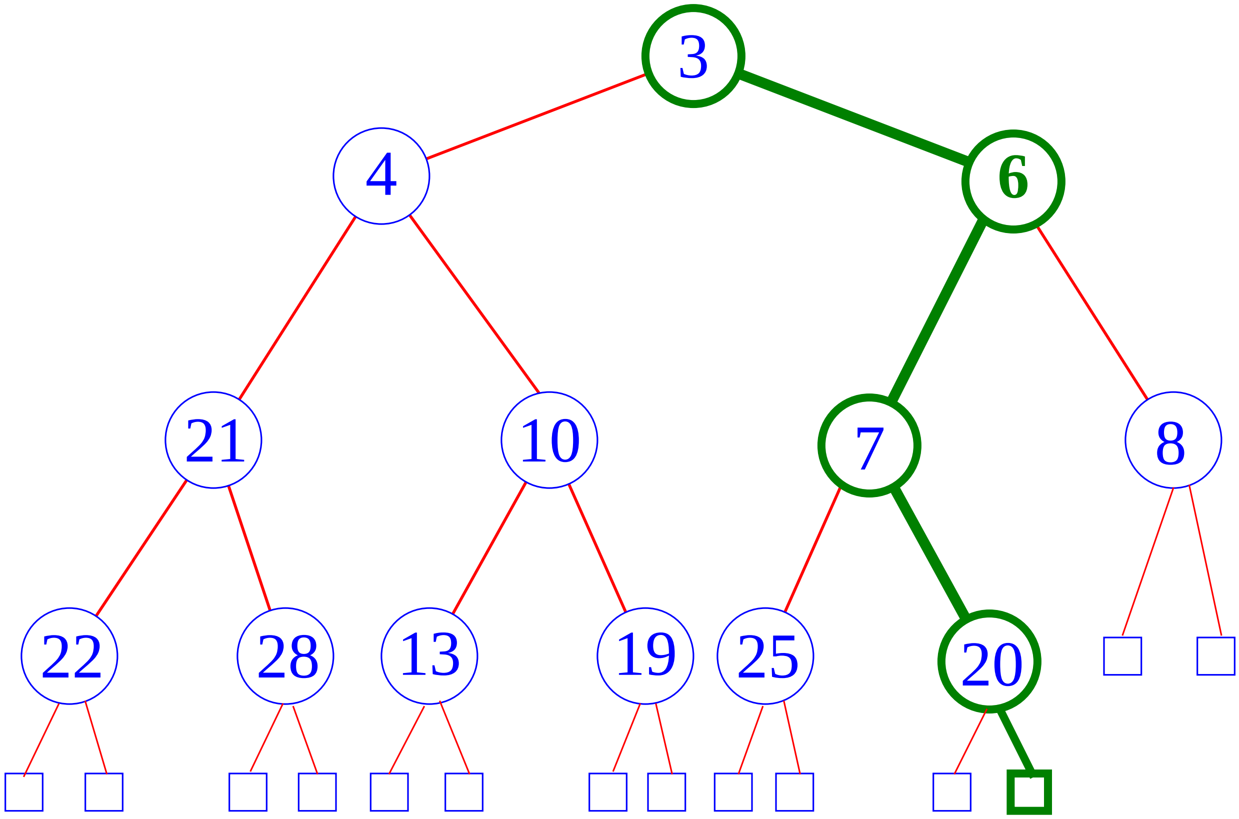



# Upheap Continues



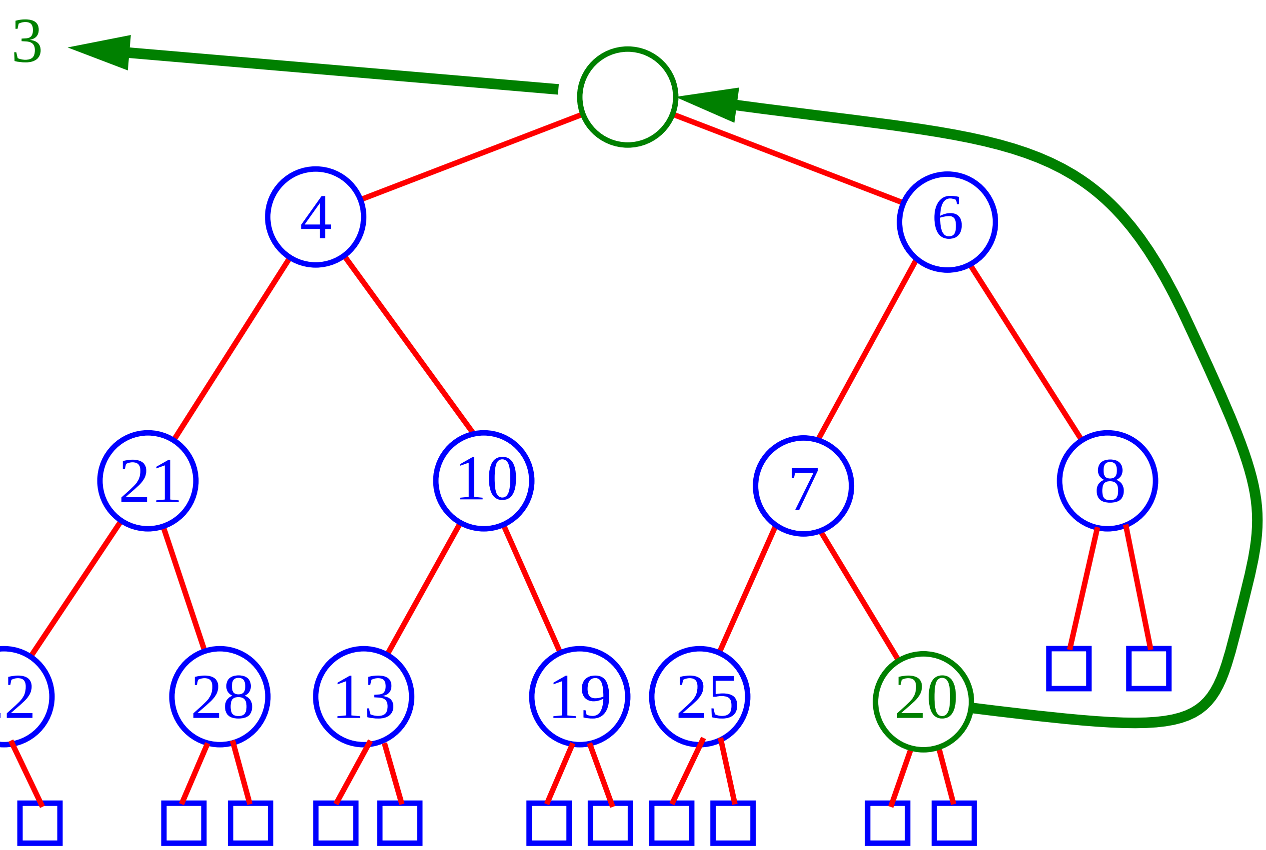
# End of Upheap




- ***Upheap*** terminates when new key is greater than the key of its parent **or** the top of the heap is reached
- (total #swaps) $\leq$ $(h - 1)$, which is O(log $n$)

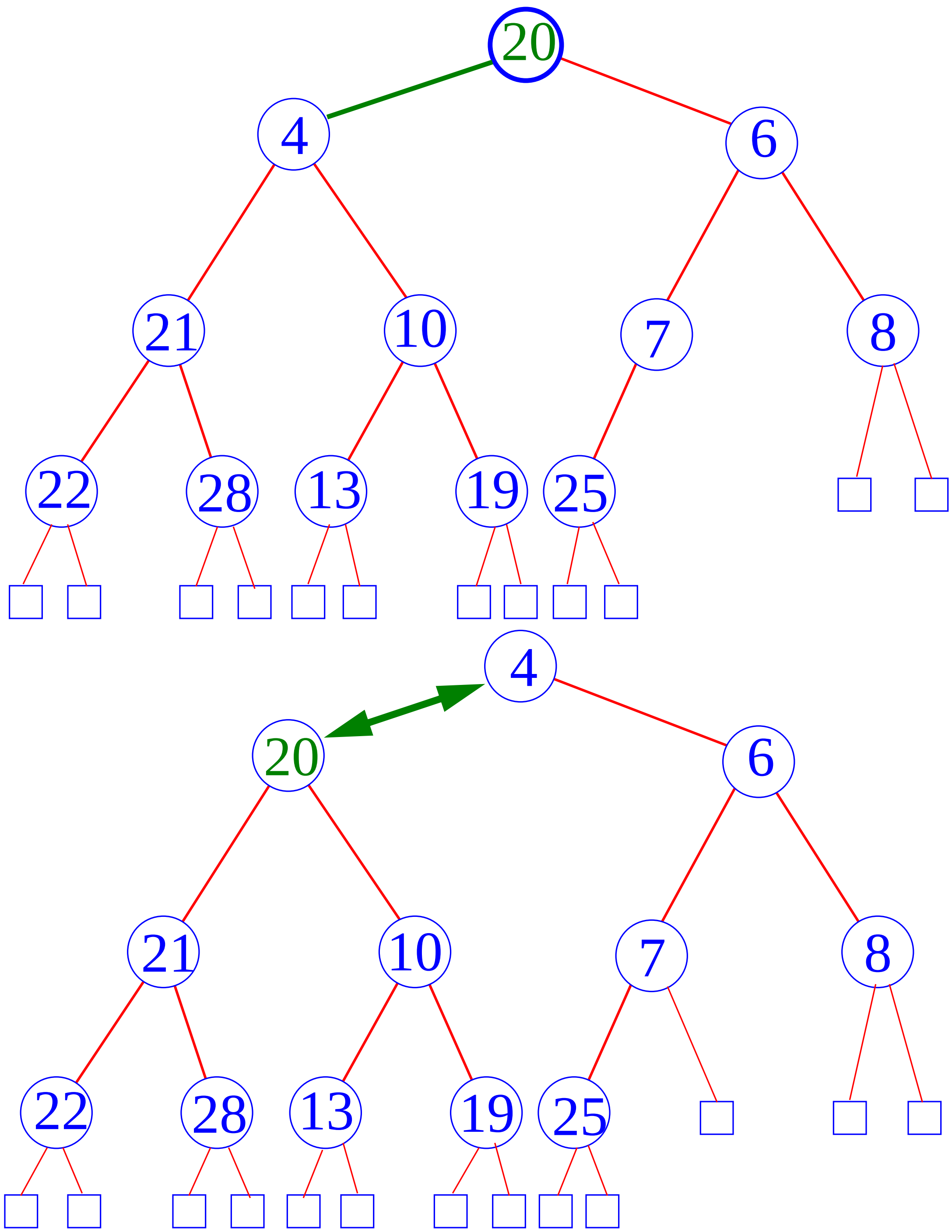
# Removal From a Heap

## RemoveMin()



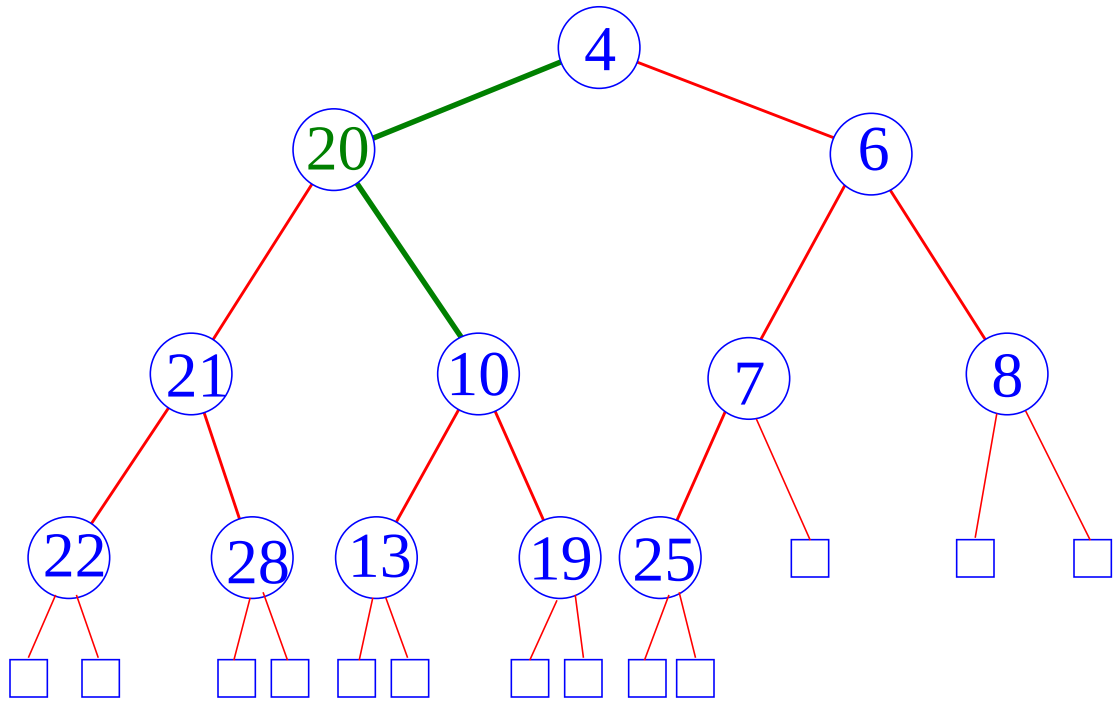

- The removal of the top key leaves a hole
- We need to fix the heap
- First, replace the hole with the last key in the heap
- Then, begin ***Downheap***

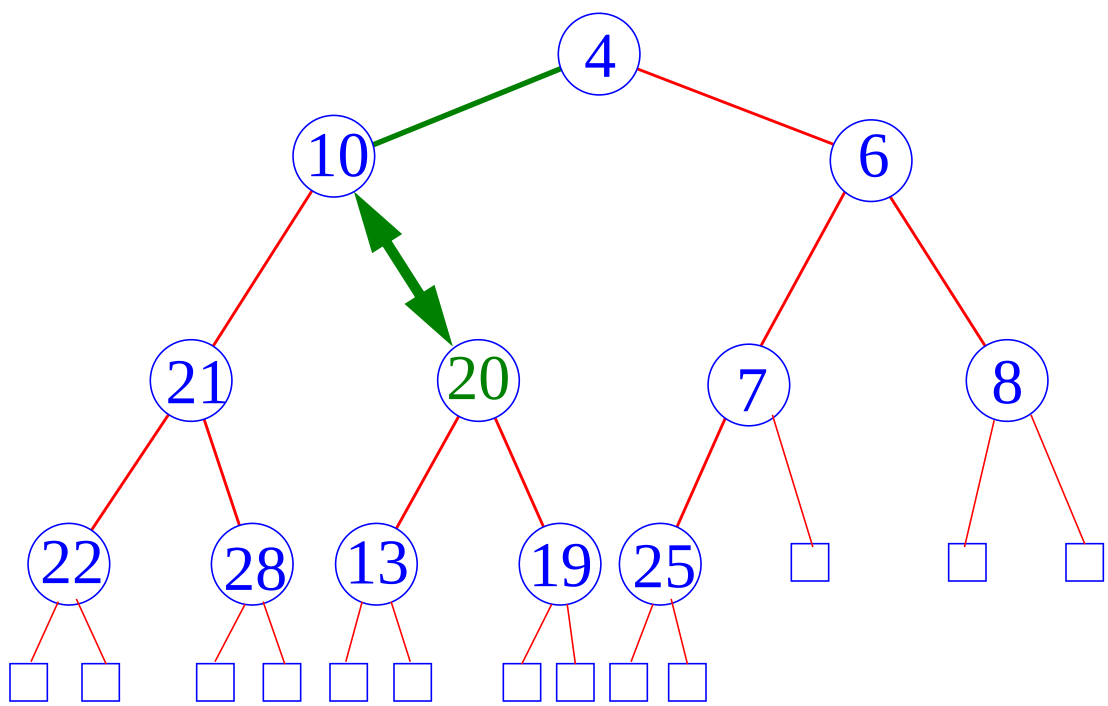
# Downheap



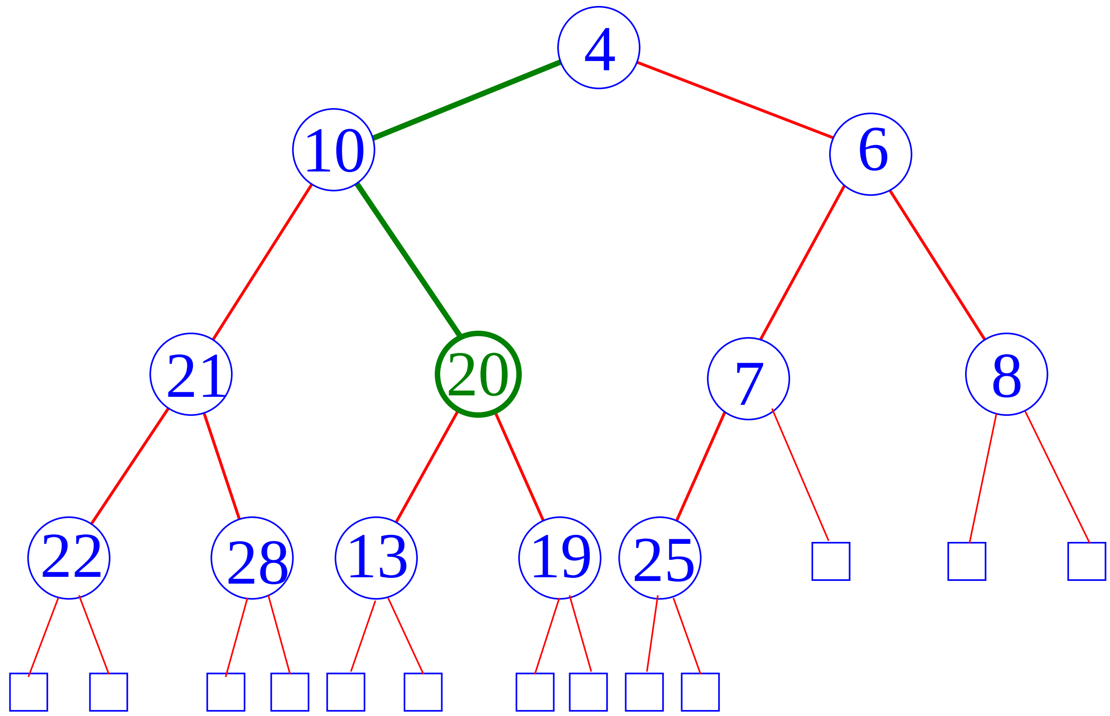

***Downheap*** compares the parent with the smallest child. If the child is smaller, it switches the two.

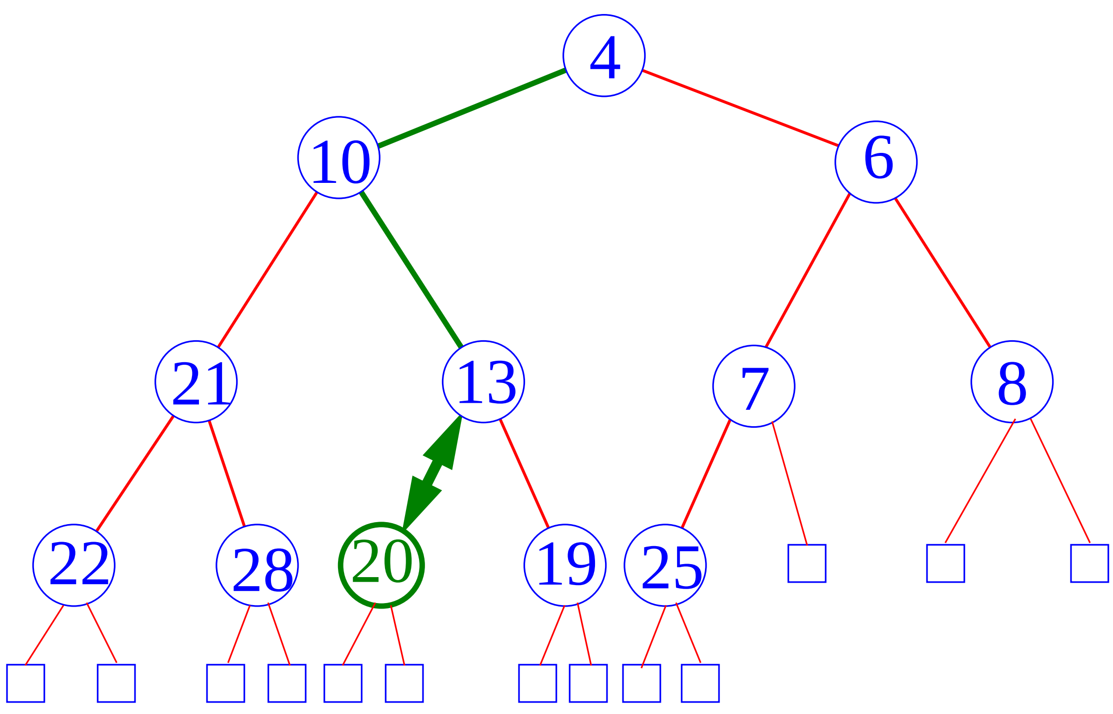
# Downheap Continues



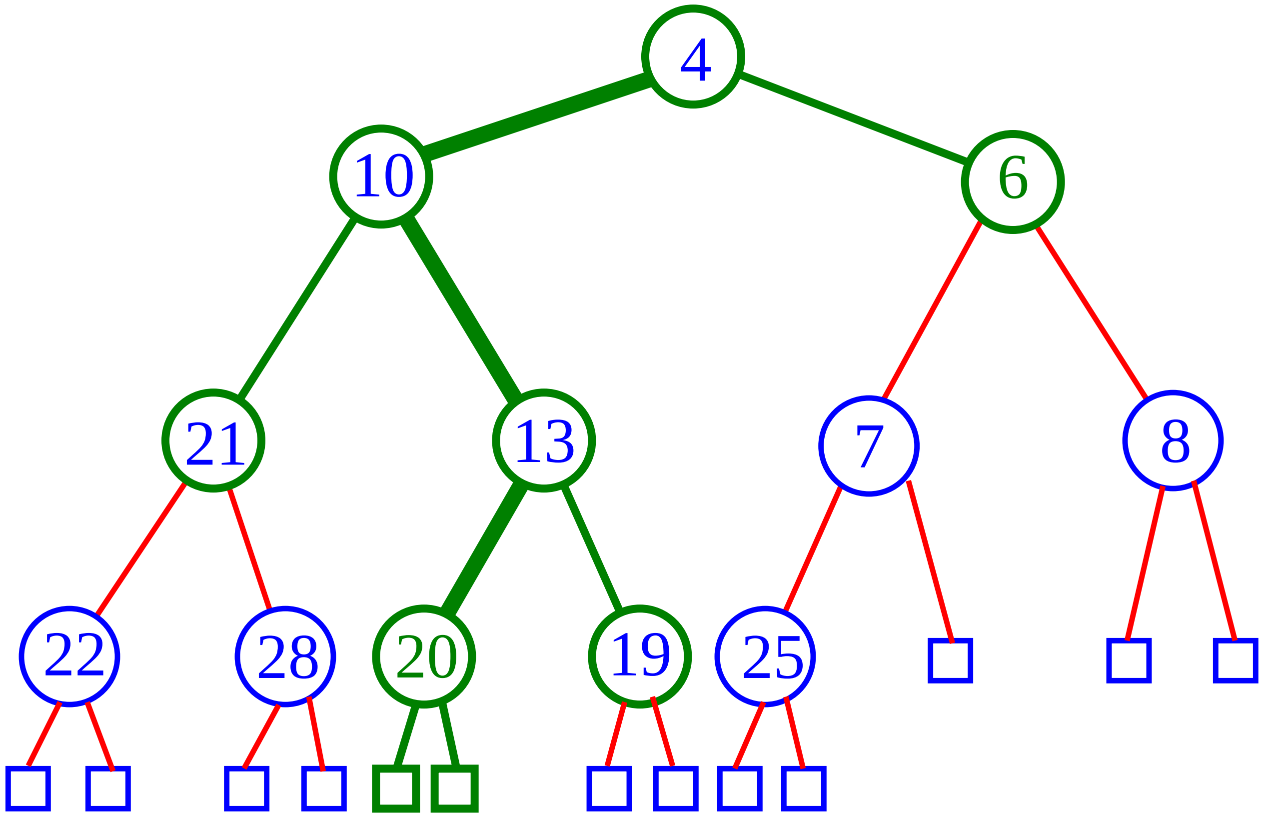



# Downheap Continues



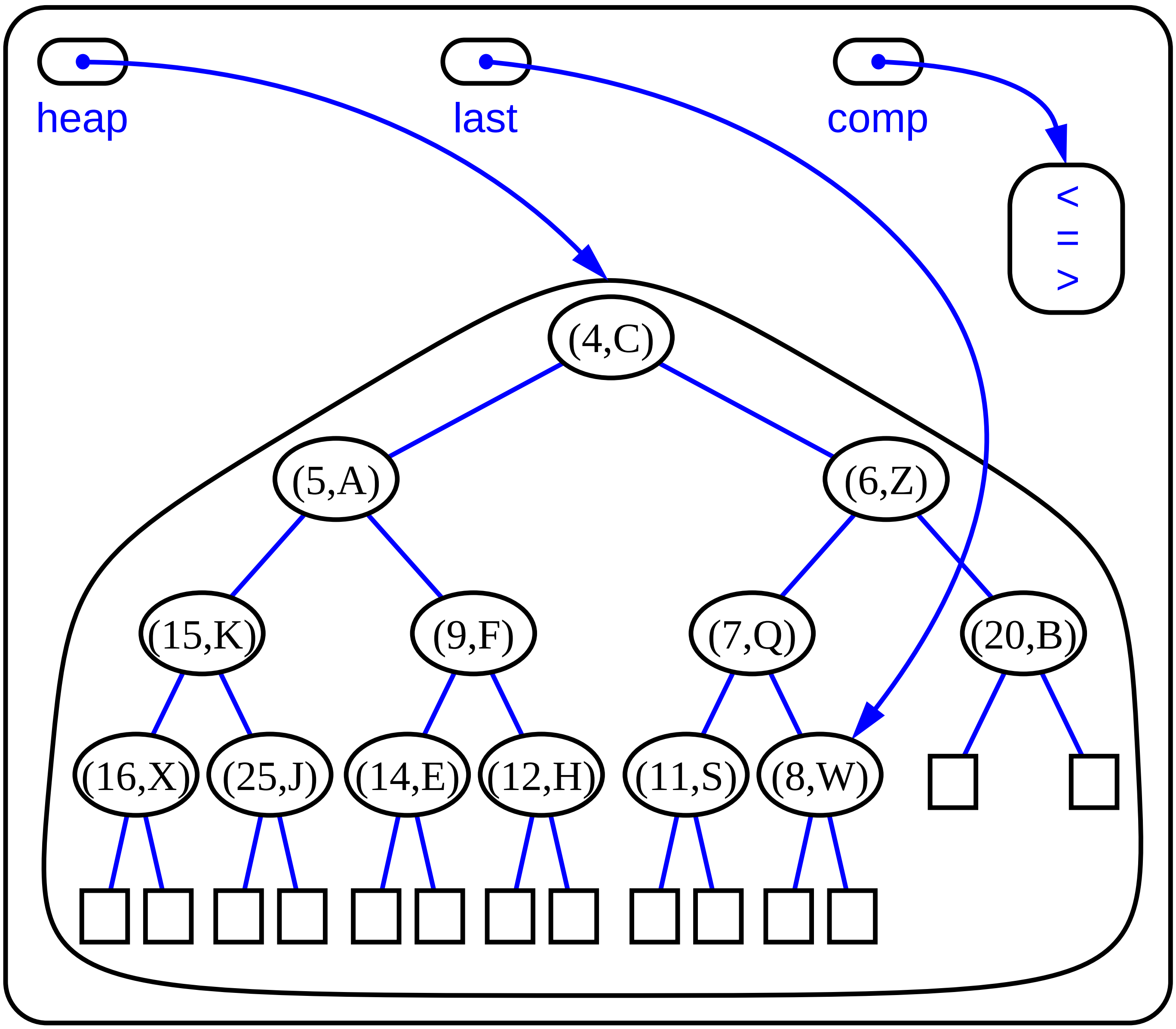
# End of Downheap




- ***Downheap*** terminates when the key is greater than the keys of both its children **or** the bottom of the heap is reached.
- (total #swaps) $\leq$ $(h - 1)$, which is O(log $n$)

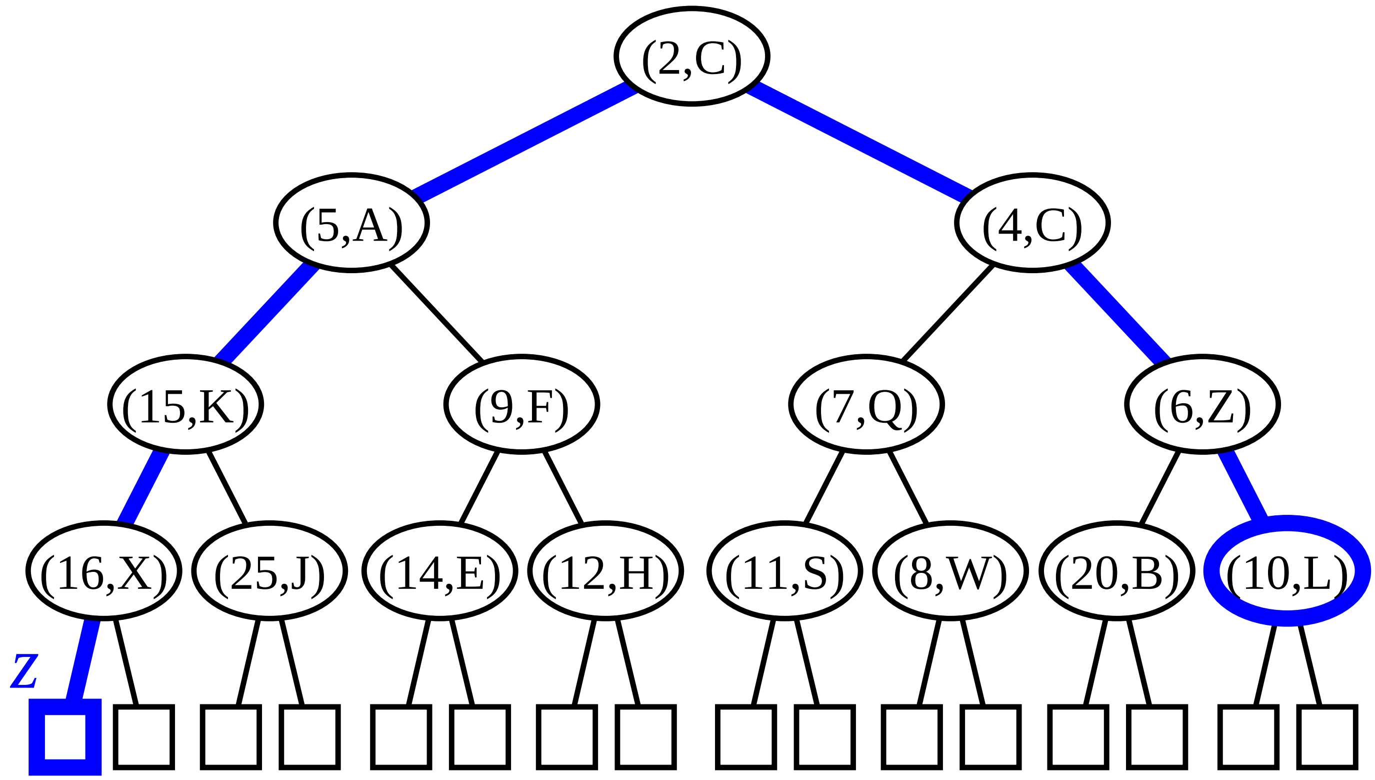
# Implementation of a Heap

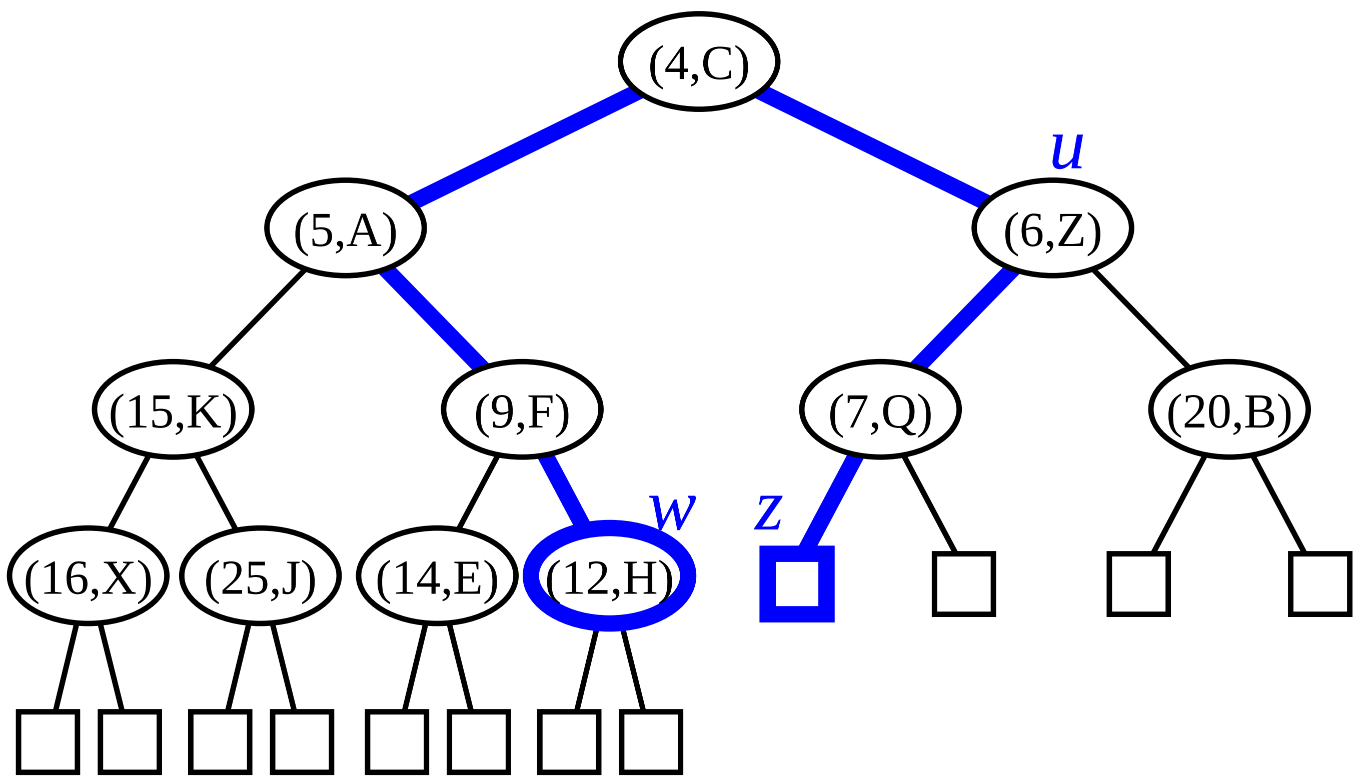
```
public class HeapPriorityQueue implements PriorityQueue
{
   BinaryTree T;
   Position last;
   Comparator comparator;
   ...
}
```



# Implementation of a Heap(cont.)

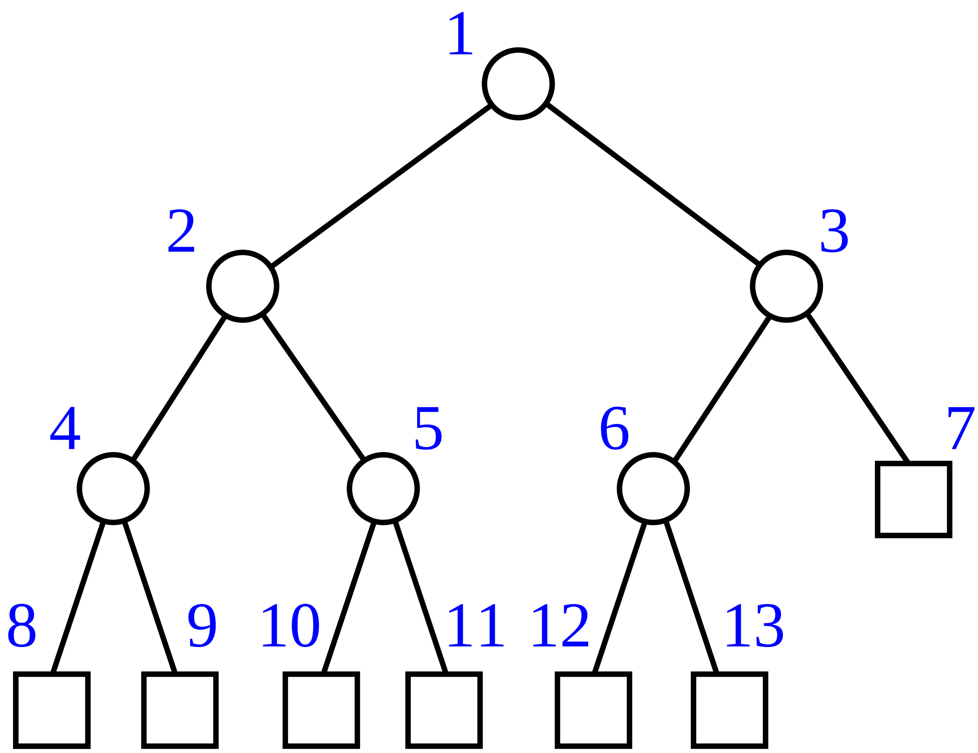
- Two ways to find the insertion position z in a heap:



# Vector Based Implementation

- Updates in the underlying tree occur only at the “last element”
- A heap can be represented by a vector, where the node at rank *i* has
  - left child at rank 2*i* and
  - right child at rank 2*i* + 1




- The leaves do no need to be explicitly stored
- Insertion and removals into/from the heap correspond to insertLast and removeLast on the vector, respectively

# Heap Sort

- All heap methods run in logarithmic time or better
- If we implement PriorityQueueSort using a heap for our priority queue, insertItem and removeMin each take O(log $k$), $k$ being the number of elements in the heap at a given time.
- We always have at most $n$ elements in the heap, so the worst case time complexity of these methods is O(log $n$).
- Thus each phase takes O($n$ log $n$) time, so the algorithm runs in O($n$ log $n$) time also.
- This sort is known as ***heap-sort***.
- The O($n$ log $n$) run time of heap-sort is much better than the O($n^2$) run time of selection and insertion sort.

# In-Place Heap-Sort

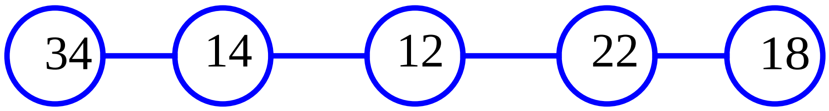
- Do not use an external heap
- Embed the heap into the sequence, using the vector representation

# The Dictionary ADT

- a dictionary is an abstract model of a database
- like a priority queue, a dictionary stores key-element pairs
- the main operation supported by a dictionary is searching by key
- simple container methods:
  - size()
  - isEmpty()
  - elements()
- query methods:
  - findElement($k$)
  - findAllElements($k$)
- update methods:
  - insertItem($k$, $e$)
  - removeElement($k$)
  - removeAllElements($k$)
- special element
  - NO_SUCH_KEY, returned by an unsuccessful search

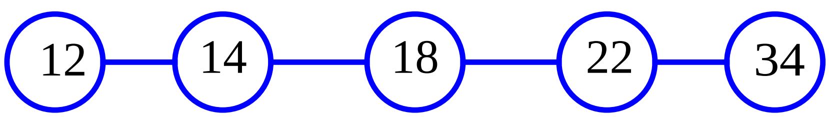
# Implementing a Dictionary with a Sequence

- ***unordered sequence***




  - searching and removing takes O($n$) time
  - inserting takes O(1) time
  - applications to log files (frequent insertions, rare searches and removals)

- ***array-based ordered sequence*** (assumes keys can be ordered)




  - searching takes O(log $n$) time (***binary search***)
  - inserting and removing takes O($n$) time
  - application to look-up tables (frequent searches, rare insertions and removals)