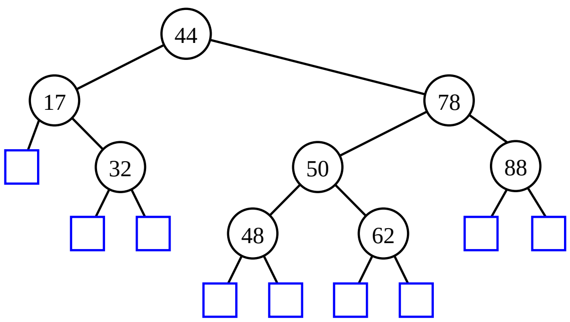
SEARCHING

- the dictionary ADT
- binary search
- binary search trees

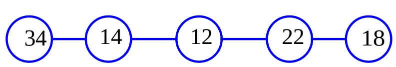


The Dictionary ADT

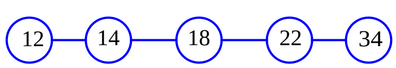
- a dictionary is an abstract model of a database
- like a priority queue, a dictionary stores key-element pairs
- the main operation supported by a dictionary is searching by key
- simple container methods:
  - size()
  - isEmpty()
  - elements()
- query methods:
  - findElement(k)
  - findAllElements(k)
- update methods:
  - insertItem(k, e)
  - removeElement(k)
  - removeAllElements(k)
- special element
  - NO_SUCH_KEY, returned by an unsuccessful search

Implementing a Dictionary with a Sequence

- ***unordered sequence***



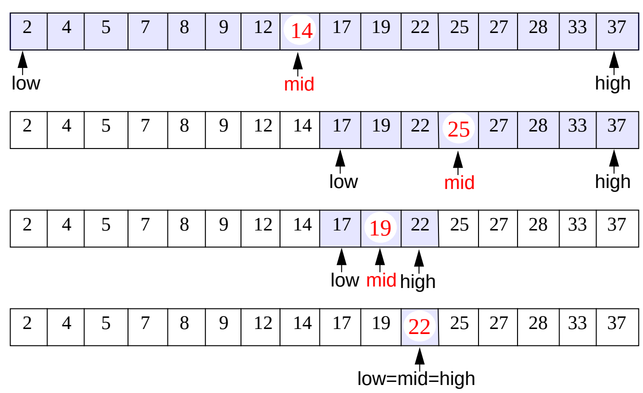
  - searching and removing takes O(n) time
  - inserting takes O(1) time
  - applications to log files (frequent insertions, rare searches and removals)
- ***array-based ordered sequence*** (assumes keys can be ordered)



  - searching takes O(log n) time (***binary search***)
  - inserting and removing takes O(n) time
  - application to look-up tables (frequent searches, rare insertions and removals)

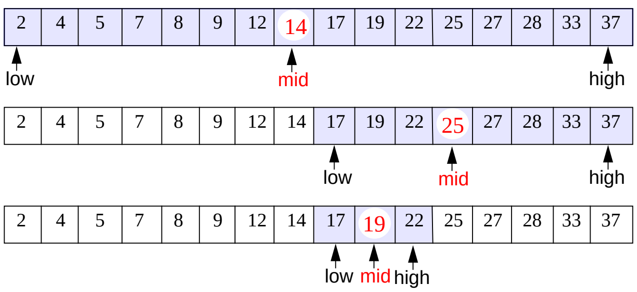
Binary Search

- narrow down the search range in stages
- “high-low” game
- findElement(22)



Pseudocode for Binary Search

Algorithm BinarySearch(S, k, low, high)
if low > high **then**
  return NO_SUCH_KEY
else
  mid ← (low+high) / 2
  if k = key(mid) **then**
    return key(mid)
  else if k < key(mid) **then**
    return BinarySearch(S, k, low, mid–1)
  else
    return BinarySearch(S, k, mid+1, high)



Running Time of Binary Search

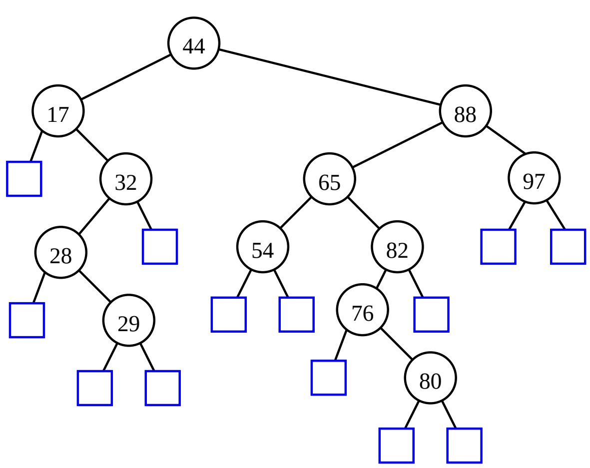
- The range of candidate items to be searched is ***halved after each comarison***

comparison	search range
0	n
1	$n/2$
2	$n/4$
…	…
2^i	$n/2^i$
$\log_2 n$	1

- In the array-based implementation, access by rank takes O(1) time, thus binary search runs in **O(log *n*)** time

Binary Search Trees

- A binary search tree is a binary tree T such that
  - each internal node stores an item (k, e) of a dictionary.
  - keys stored at nodes in the left subtree of v are less than or equal to k.
  - keys stored at nodes in the right subtree of v are greater than or equal to k.
  - kxternal nodes do not hold elements but serve as place holders.



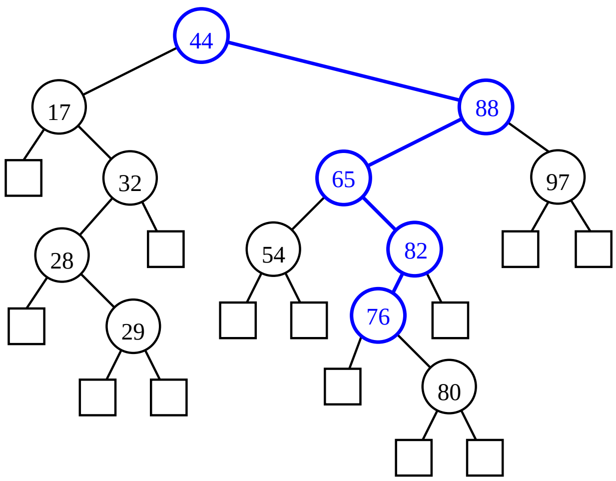
Search

- A binary search tree *T* is a ***decision tree***, where the question asked at an internal node *v* is whether the search key *k* is less than, equal to, or greater than the key stored at *v*.
- Pseudocode:

Algorithm TreeSearch(*k*, *v*):
  Input: A search key *k* and a node *v* of a binary search tree *T*.
  Ouput: A node w of the subtree *T*(*v*) of *T* rooted at *v*, such that either w is an internal node storing key *k* or w is the external node encountered in the inorder traversal of *T*(*v*) after all the inter nal nodes with keys smaller than *k* and before all the internal nodes with keys greater than *k*.
  if *v* is an external node **then**
    return *v*
  if *k* = key(*v*) **then**
    return *v*
  else if *k* < key(*v*) **then**
    return TreeSearch(*k*, *T*.leftChild(*v*))
  else
    { *k* > key(*v*) }
    return TreeSearch(*k*, *T*.rightChild(*v*))

Search Example I

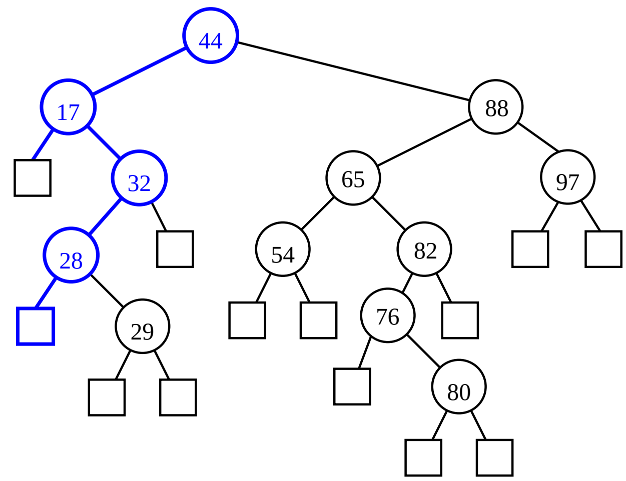
- Successful findElement(76)



- A successful search traverses a path starting at the root and ending at an internal node
- How about findAllelements(*k*)?

Search Example II

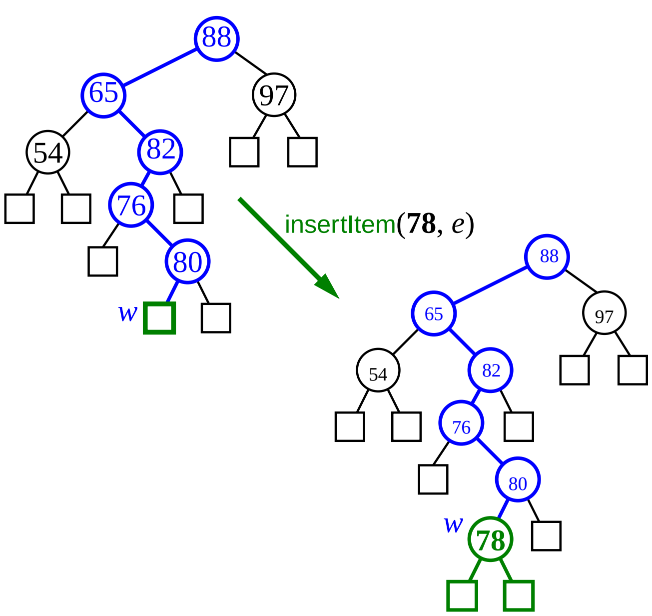
- Unsuccessful findElement(25)



- An unsuccessful search traverses a path starting at the root and ending at an external node

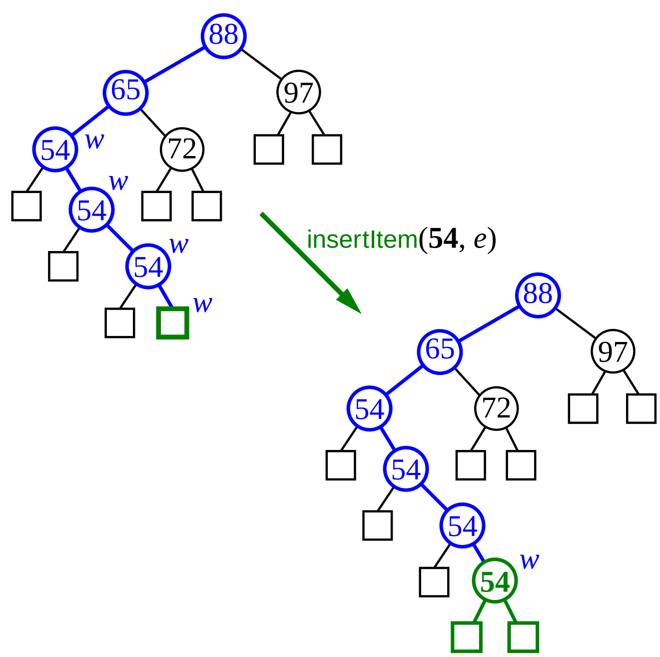
Insertion

- To perform insertItem(*k*, *e*), let *w* be the node returned by TreeSearch(*k*, *T*.root())
- If *w* is external, we know that *k* is not stored in *T*. We call expandExternal(*w*) on *T* and store (*k*, *e*) in *w*



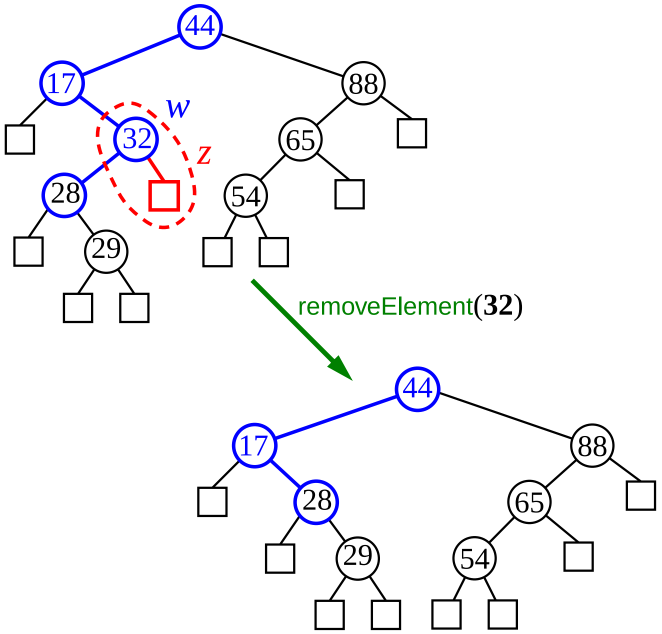
Insertion II

- If *w* is internal, we know another item with key *k* is stored at *w*. We call the algorithm recursively starting at *T*.rightChild(*w*) or *T*.leftChild(*w*)



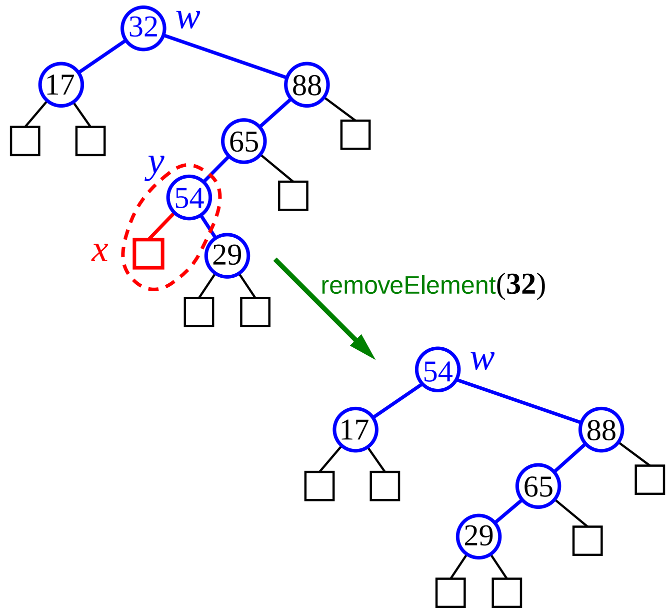
Removal I

- We locate the node *w* where the key is stored with algorithm TreeSearch
- If *w* ***has an external child*** *z*, we remove *w* and *z* with removeAboveExternal(*z*)



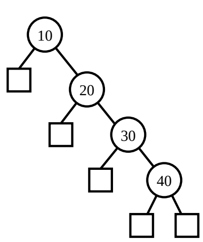
Removal II

- If *w* ***has an no external children***:
  - find the internal node *y* following *w* in inorder
  - move the item at *y* into *w*
  - perform removeAboveExternal(*x*), where *x* is the left child of *y* (guaranteed to be external)



Time Complexity

- A search, insertion, or removal, visits the nodes along a ***root-to leaf path***, plus possibly the ***siblings*** of such nodes
- Time O(1) is spent at each node
- The running time of each operation is O(*h*), where *h* is the height of the tree
- The height of binary serch tree is in *n* in the worst case, where a binary search tree looks like a sorted sequence



- To achive good running time, we need to keep the tree ***balanced***, i.e., with O(log *n*) height
- Various balancing schemes will be explored in the next lectures