COSC 2011 Section N

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Overview

- Skip Lists
  - Description, Definition
  - ADT
  - Searching
  - Insertion
  - Removal
  - Notes

Skip Lists - Description: (1)

- Data structure for efficient realization of the *Ordered Dictionary*.
- Makes random choices in arranging the items
  - ***Average*** Search and Update times: O(log n)
  - Doesn't depend on the probability distribution of the keys!

Skip Lists - Description: (2)

- Random Number Generator for Insertions:
  - Help decide where to place a new item.
  - Data structures and algorithms utilizing *randomization* are usually simple and efficient!

Skip Lists – Random Numbers (1)

- Extensive use for Random Numbers:
  - Cryptography, computer simulations, computer games…
  - Usually, not really random! But rather, ***pseudo-random.***
    - Generates random-like numbers.
    - Good enough for most situations!

Skip Lists – Random Numbers (2)

- Random Numbers with Java:
  - Math.random()
  - Java.util.Random()

Skip Lists – Definition: (1)

- *Skip List* ***S*** for Dictionary ***D***:
  - Consists of a series of lists $\{S_0, S_1, S_2, \ldots, S_h\}$.
  - Each list S_i stores a subset of the items of *D*:
    - ⋆ Sorted by non - decreasing key
    - ⋆ Items with two special keys: **+¥** and **-¥**
    - ⋆ **-∞ ®** less than every possible key k to be inserted in *D*.

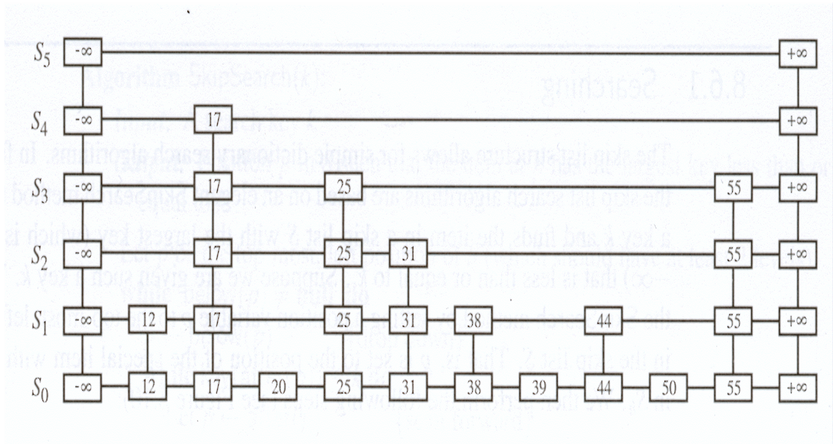
Skip Lists – Definition: (2)

- ⋆ **+∞ ®** greater than every possible key k to be inserted in *D*.
- Each S_i also satisfies the following:

1. List S_0 contains every item of *D* and -∞, +∞
2. For i = 1 .. *h*, S_i contains a random generated subset of items in List S_{i-1} & -∞, +∞
3. List Sh contains only -∞, +∞

Skip Lists – Definition: (3)

- 2D collection of *positions* arranged horizontally into ***levels*** and vertically into ***towers***.



Skip Lists – Definition: (4)

- Explanation:
  - S_{I+1} contains approx. every other item in S_i.
  - Items in S_{i+1} are chosen randomly from S_i.
    - ⋆ Probability of ½ for each item.
  - In general: S_i contains approx. $n / 2^i$ items.
    - ⋆ Height $h \rightarrow \log n$

Skip Lists – ADT

- ADT Specialized Methods:

after(p): Return position after *p* on same level.

before(p): Return position before *p* on same level.

below(p): Return position below *p* on same tower.

above(p): Return position above *p* on same tower.

Return **null** if no position!

Skip Lists – Searching: (1)

- *SkipSearch* Algorithm:

Input: Search key k

Output: Position p in S s.t item at p has largest key ≤ k

Let p be top most, left position of S:

```
while below(p) ≠ null do
    p ← below(p)    {drop down}
    while key (after(p)) ≤ k do
        p ← after(p)    {scan forward}
return p
```


Skip Lists – Searching: (2)

- *SkipSearch* Algorithm:
  - Takes key k and finds item in S with largest key ≤ k (possibly -∞) .
  - Begin at top most, left position in S (-∞) call it *p*:

1. If S.below(*p*) = null then done – located largest key ≤ k in S. otherwise drop down a level: $p \leftarrow$ below(*p*).

Skip Lists – Searching: (2)

2. Otherwise, if S.after(p) ≤ k then p ← after(p). When after(p) > key, go back to step 1.

Skip Lists – Insertion: (1)

- Insertion:
  - Randomization to decide how many references to new item (k, e) to add.
  - Perform SkipSearch(k)
    - Gives position p of bottom level item with key ≤ k.
  - Insert new item right after p on same level.

Skip Lists – Insertion: (2)

- Call method random - returns number between 0 and 1.
  - If return is ≤ ½ add copy of new item one level up.
  - If return > ½ stop – do not insert any more copies!
  - This process creates the tower.

Skip Lists – Insertion: (3)

Algorithm SkipInsert(k, e)

Input: Item(k, e)
Output: None

```
p ← SkipSearch(k)
q ← insertAfterAbove(p, null, (k,e))
while random() < ½ do
    while above(p) = null do
        p ← before(p)
    p ← above(p)
    q ← insertAfterAbove(p, q, (k,e))
```

Skip Lists – Removal: (1)

Algorithm SkipRemove(k)

Input: Key k

Output: Position (element with key k) if found otherwise NO_SUCH_KEY

$p \leftarrow$ SkipSearch(k)
if key(p) $\neq$ k
return NO_SUCH_KEY
else
use *above*(p) to go to top most level of position p in tower
remove all positions in tower starting from top

Skip Lists –Notes: (1)

- Insert / Removal: O(log n)
- Improvements:
  - No need for references to the items at levels above 0 – only keys are needed.
  - No need for *above* and *before* methods.
    - ★ Perform insertion and removal in a top-down scan-forward approach thus saving space for “up”, “prev” references

Skip Lists –Notes: (2)

- Improvements wont affect running time by more than a constant!
- Experiments indicate optimized Skip Lists in practice are faster than AVL trees and other balanced search trees!
- Keep reference to top most left position in S.
  - Can’t insert beyond top level of S.