ELIC 629: Digital Image Processing
Winter 2005
Instructor: Bill Kapralos

Lab 2 Assignment Questions – Solutions

Problem 2.2
Brightness adaptation.

Problem 2.14
A region R of an image is composed of a set of connected points in an image. The boundary of an image is the set of points that have one or more neighbors that are not in R. Because boundary points also are part of R, it follows that a point on the boundary has at least one neighbor in R and at least one neighbor not in R. (If the point in the boundary did not have a neighbor in R, the point would be disconnected from R, which violates the definition of points in a region). Since all points in R are part of a connected component, all point sin the boundary are also connected and a path (entirely in R) exists between any two points on the boundary. Thus the boundary forms a closed path.

Problem 2.19
The median ξ of a set of numbers is such that half the values in the set are below ξ and the other half are above it. A simple example will suffice to show that Eq. 2.6-1 is violated by the median operator. Let S1 = {1, -2, 3}, S2 = {4,5,6} and a = b = 1. In this case H is the median operator. We then have H(S1 + S2) = median {5,3,9} = 5, where it is understood that S1 + S2 is the element-by-corresponding-element sum of S1 and S2. Next, we compute H(S1) = median{1,-2,3} = 1 H{S2} = median{4,5,6} = 5. Then since H(aS1 +bS2) does not equal aH(S1) + bH(S2), it follows that Eq. 2.6-1 is violated and the median operator is non-linear.

Problem 3.1

(a) General form: $s = T(r) = Ae^{-Kr^2}$. For the condition shown in the problem figure, $Ae^{-KL_0^2} = A/2$. Solving for K yields

$$\begin{aligned} -KL_0^2 &= \ln(0.5) \\ K &= 0.693/L_0^2. \end{aligned}$$

Then,

$$s = T(r) = Ae^{-\frac{0.693}{L_0^2}r^2}.$$

(b) General form: $s = T(r) = B(1 - e^{-Kr^2})$. For the condition shown in the problem figure, $B(1 - e^{-KL_0^2}) = B/2$. The solution for K is the same as in (a), so

$$s = T(r) = B(1 - e^{-\frac{0.693}{L_0^2}r^2})$$

(c) General form: $s = T(r) = (D - C)(1 - e^{-Kr^2}) + C$.

Problem 3.12

The purpose of this simple problem is to make the student think of the meaning of histograms and arrive at the conclusion that histograms carry no information about spatial properties of images. Thus, the only time that the histogram of the images formed by the operations shown in the problem statement can be determined in terms of the original histograms is when one or both of the images is (are) constant. In (d) we have the additional requirement that none of the pixels of $g(x, y)$ can be 0. Assume for convenience that the histograms are not normalized, so that, for example, $h_f(r_k)$ is the number of pixels in $f(x, y)$ having gray level r_k, assume that all the pixels in $g(x, y)$ have constant value c. The pixels of both images are assumed to be positive. Finally, let u_k denote the gray levels of the pixels of the images formed by any of the arithmetic operations given in the problem statement. Under the preceding set of conditions, the histograms are determined as follows:

(a) The histogram $h_{\text{sum}}(u_k)$ of the sum is obtained by letting $u_k = r_k + c$, and $h_{\text{sum}}(u_k) = h_f(r_k)$ for all k. In other words, the values (height) of the components of h_{sum} are the same as the components of h_f, but their locations on the gray axis are shifted right by an amount c.

(b) Similarly, the histogram $h_{\text{diff}}(u_k)$ of the difference has the same components as h_f but their locations are moved left by an amount c as a result of the subtraction operation.

(c) Following the same reasoning, the values (heights) of the components of histogram $h_{\text{prod}}(u_k)$ of the product are the same as h_f, but their locations are at $u_k = c \times r_k$. Note that while the spacing between components of the resulting histograms in (a) and (b) was not affected, the spacing between components of $h_{\text{prod}}(u_k)$ will be spread out by an amount c.

(d) Finally, assuming that $c \neq 0$, the components of $h_{\text{div}}(u_k)$ are the same as those of h_f, but their locations will be at $u_k = r_k/c$. Thus, the spacing between components of $h_{\text{div}}(u_k)$ will be compressed by an amount equal to $1/c$.

The preceding solutions are applicable if image $f(x,\, y)$ also is constant. In this case the four histograms just discussed would each have only one component. Their location would be affected as described (a) through (c).

Problem 3.13

Using 10 bits (with one bit being the sign bit) allows numbers in the range -511 to 511. The process of repeated subtractions can be expressed as

$$\begin{aligned} d_K(x,y) &= a(x,y) - \sum_{k=1}^{K} b(x,y) \\ &= a(x,y) - K \times b(x,y) \end{aligned}$$

where K is the largest value such that $d_K(x,y)$ does not exceed -511 at any coordinates (x,y), at which time the subtraction process stops. We know nothing about the images, only that both have values ranging from 0 to 255. Therefore, all we can determine are the maximum and minimum number of times that the subtraction can be carried out and the possible range of gray-level values in each of these two situations.

Because it is given that $g(x,y)$ has at least one pixel valued 255, the maximum value that K can have before the subtraction exceeds -511 is 3. This condition occurs when, at some pair of coordinates (s,t), $a(s,t) = b(s,t) = 255$. In this case, the possible range of values in the difference image is -510 to 255. The latter condition can occur if, at some pair of coordinates (i,j), $a(i,j) = 255$ and $b(i,j) = 0$.

The minimum value that K will have is 2, which occurs when, at some pair of coordinates, $a(s,t) = 0$ and $b(s,t) = 255$. In this case, the possible range of values in the difference image again is -510 to 255. The latter condition can occur if, at some pair of coordinates (i,j), $a(i,j) = 255$ and $b(i,j) = 0$.