ELIC 629
Digital Image Processing

Fall 2005

Image Enhancement in the Spatial Domain:
Basics of Spatial Filtering, Smoothing Spatial Filters, Sharpening Spatial Filters

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Overview (1):

- **Before We Begin**
  - Administrative details
  - Review → some questions to consider
- **Spatial Filtering**
  - Brief review
- **Smoothing Spatial Filters**
  - Introduction
  - Smoothing linear filters
  - Order statistics filters

Overview (2):

- **Introduction to Sharpening Filters**
  - Purpose
  - Foundation
  - Introduction to digital derivatives

Before We Begin

Administrative Details (1):

- **Lab Two**
  - Lab reports have been graded and will be returned during the lab period
  - Grade break-down (/40)
    - Attendance → 10
    - Report → 20
    - Assignment → 10

Administrative Details (2):

- **Lab Two**
  - Lab reports have been graded and will be returned during the lab period.
- **Lab Five**
  - Covers material we have not covered yet and some material we may not cover at all but should still be a fun and interesting lab!
  - Lab report required for this lab
  - No assignment

Administrative Details (3):

- **Mid-Term Exam**
  - November 7 2005
  - 6:05pm – ??? (won't be the entire period but you will have enough time) → be here on time!
  - Review for test will be Oct. 31 during the lab period in the lab (N210) ~ 1 hour
  - I will post questions for you to work on (not to be handed in)
  - Work on questions from the book, especially ones with solutions! (see book website for solutions to some questions)

Some Questions to Consider (1):

- Describe the mechanics of spatial filtering

Basics of Spatial Filtering – Review

Introduction – 2D Masks (1):

- **Graphical Illustration**

Mask (kernel, template…) with its coefficients

Pixels of sub-image under mask

Introduction – 2D Masks (2):

- **Template, Kernel, Mask**
  - Coefficients denoted by w
  - Origin is in the "middle"
  - Arbitrary size → dimensions do not need to be odd implying no "true" center (origin)

w(-1,-1)	w(-1,0)	w(-1,1)
w(0,-1)	w(0,0)	w(0,1)
w(1,-1)	w(-1,0)	w(1,1)

Example of a 3x3 template with its coefficients

Introduction – 2D Masks (3):

- **"Mechanics" of Spatial Filtering**
  - Moving the template over each pixel of the image
    - At each pixel (x,y) the response (e.g., output value) is determined using some pre-defined relationship
    - For linear spatial filtering, response is given by a sum of the products of the filter coefficients (denoted by w) and the corresponding image pixels "under" the area of the template
    - Mathematically, response g at (x,y) given as

$$g(x,y) = w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,y) + \ldots + w(0,0)f(x,y) + \ldots + w(1,0)f(x+1,y) + w(1,1)f(x+1,y+1)$$

Introduction 2D – Masks (4):

- **"Mechanics" of Spatial Filtering (cont…)**
  - General filtering expression

$$g(x,y) \quad = \quad \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s,y+t)$$

  - m → row dimension
  - n → column dimension
  - m = 2a+1 and n = 2b+1 and a,b are non-negative integers or a = (m-1)/2 and b = (n-1)/2
  - But this gives output for one pixel location (x,y) only!

Introduction - 2D Masks (5):

- **"Mechanics" of Spatial Filtering (cont…)**
  - To generate complete output image, the above equation (process) must be applied to each pixel of input image e.g., for each x = 0 – M-1 and y = 0 – N-1 where M,N are the number of rows and columns of the input image
  - Similar to a frequency domain concept called convolution (more on this in the future…)
    - Hence sometimes referred to as "*convolving a mask with an image*" and the mask is often called a convolution mask

Smoothing Spatial Filters

Introduction (1):

- **Purpose of Smoothing Spatial Filters**
  - Used for blurring, particularly in pre-processing
    - Removal of small detail prior to extraction of large object(s) in image
    - Bridging ("closing") of small gaps in lines or curves
  - Also used for noise reduction
    - Can be achieved with a linear or non-linear filter

Smoothing Linear Filters (1):

- **Essentially an Averaging Filter**
  - Output of smoothing filter is simply the average of pixels in the neighborhood of filter mask (kernel)
  - Also known as a low pass filter
    - Eliminates high frequency components (we will describe this further in later lectures)
  - Idea of smoothing filter
    - Random noise typically consists of sharp transitions in gray levels

Smoothing Linear Filters (2):

- Idea of smoothing filter (cont…)
  - By replacing the value of every pixel by the average of itself and its neighbors, we essentially reduce the sharp transitions in gray levels
- Most obvious application of a smoothing filter is noise reduction
- However, beware!
  - Not all sharp transitions are bad and un-wanted!
  - Edges which are typically very important and wanted features of an image are also defined as sharp transitions in gray levels

Smoothing Linear Filters (3):

- However, beware! (cont…)
  - Since smoothing filter removes sharp transitions in gray level, averaging (smoothing) filters blur edges!
- Several other applications of smoothing (averaging) filters in addition to noise reduction
  - Smoothing of false contours which result when not using a large number of gray levels
  - Removing irrelevant details in an image → pixel regions that are small in comparison to the size of the filter kernel

Smoothing Linear Filters (4):

- **Smoothing Filter Kernels**
  - Example of a 3 x 3 smoothing filter

$$\frac{1}{9}\times\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} \qquad R = \frac{1}{9}\sum_{i=1}^{9} z_i$$

  - Above kernel (filter) produces average of the pixels under the mask
  - Notice that coefficients are equal to 1 and not 1/9!
    - One division instead of nine → more efficient!

Smoothing Linear Filters (5):

- **Smoothing Filter Kernels (cont…)**
  - Another example of a 3 x 3 smoothing filter

$$\frac{1}{16}\times\begin{bmatrix}1 & 2 & 1\\ 2 & 4 & 2\\ 1 & 2 & 1\end{bmatrix} \qquad R = \frac{1}{16}\sum_{i=1}^{9} z_i$$

  - This is an example of a weighted average filter
    - Coefficients are not all the same value!
    - Some pixels multiplied by higher values thus giving those pixels more importance in average

Smoothing Linear Filters (6):

- Another example of a 3 x 3 smoothing filter (cont…)

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

- Center coefficient is highest meaning center pixel is given most importance

- Other coefficients are reduced inversely as a function of distance from the center coefficient
  - Diagonal terms are further away than the "edge" neighbors and thus the corresponding pixels in image provide less importance to average

Smoothing Linear Filters (7):

- Another example of a 3 x 3 smoothing filter (cont…)
  - Many other types of coefficient masks are also available depending on application but typically try to keep the sum of the coefficients an integral power of 2 (e.g., 16 as in previous example)
- In general, hard to notice differences between images filtered by both these filter examples

Smoothing Linear Filters (8):

- **General Filter Implementation**
  - Recall the filtering expression for filtering M x N image with weighted averaging filter of size m x n

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

  - The general expression equation can now be stated as (again, given an N x M image with m x n filter where m,n are odd)

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t) f(x+s, y+t)}{\sum_{s=-a}^{a}\sum_{s=-b}^{b} w(s,t)}$$

Smoothing Linear Filters (9):

- **General Filter Implementation (cont…)**

$$g(x,y) = \frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

  - Complete filtered image is obtained by applying above equation for each x = 0,1,2, …, M-1 and y = 0,1,2, …, N-1
  - Denominator is sum of the mask (kernel) coefficients and is constant (e.g., computed once!)
    - Typically division applied once to output image rather than at each stage (pixel output)

Smoothing Linear Filters (10):

- **Graphical Examples**

Original image

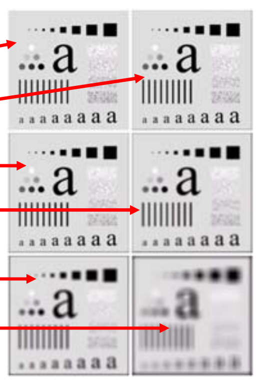
3 x 3 filter

5 x 5 filter

9 x 9 filter

15 x 15 filter

35 x 35 filter



Smoothing Linear Filters (11):

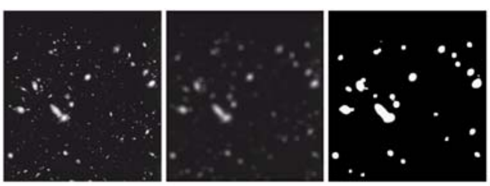
- **Some Notes Regarding Averaging Filters**
  - For small filter (e.g., 3 x 3) a slight general blurring of entire image occurs but details that are about same size of filter are affected considerably
  - Noise is less pronounced
  - Jagged borders are "pleasantly" smoothed
  - As filter size increases, blurring is more pronounced

Smoothing Linear Filters (12):

- **Image Blurring**
  - Important application of averaging filter is blurring to get "gross representation" of objects of interest
    - Smaller objects blend into the background
    - Larger objects become more "blob-like" and easy to detect
    - Size of mask (kernel) determines size of objects to be blended into background → smaller mask, smaller objects blended to background

Smoothing Linear Filters (13):

- **Image Blurring Graphical Example**
  - Image obtained with Hubble space telescope



Original image | Filtered with 15 x 15 averaging mask | Threshold image – small objects have disappeared

Order-Statistics Filters (1):

- **Non-linear Spatial Filters**
  - Response of filter is based on ordering (ranking) the pixels contained in image area encompassed by filter and then replacing center pixel with value determined by ranking result
  - One of the most known examples is the median filter

Order-Statistics Filters (2):

- **Median Filter**
  - Replaces the value of a pixel with median of the gray levels in the neighborhood of that pixel
  - Can provide excellent noise reduction with less blurring than averaging filters
    - Particularly effective for impulsive noise also known as salt-and-pepper noise due to its appearance as white and black dots in the image
  - What is the median ?
    - Given a set of values, the median ξ of the set is chosen such that half the values of set are less than ξ and half are more

Order-Statistics Filters (3):

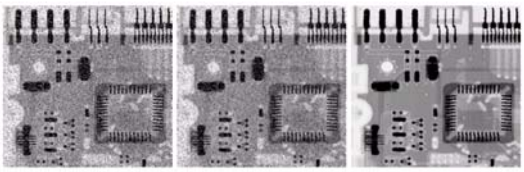
- **Median Filter (cont…)**
  - To perform median filtering:
    1. Sort the values of the pixel in question at spatial location (x,y) and its neighbors
    2. Determine the median value ξ
    3. Assign intensity value at location (x,y) the median value
  - Given 3 x 3 mask $\rightarrow$ median value is the 5^{th} largest value, 5 x 5 mask $\rightarrow$ median is 13^{th} largest value

Order-Statistics Filters (4):

- **Median Filter (cont…)**
  - Principle function of median filter is to force points with distinct gray levels to be more like their neighbors
    - Isolated clusters of pixels that are light or dark with respect to their neighbors are eliminated

Order-Statistics Filters (5):

- **Median Filter Graphical Example**
  - Comparison between averaging filter



Original image corrupted by salt-and-pepper noise | Image filtered with 3 x 3 average filter | Image filtered with 3 x 3 median filter

Sharpening Spatial Filters

Introduction (1):

- **Principle Objective of Sharpening Filters**
  - Highlight fine detail in an image
  - Enhance detail that has been blurred either in error or as a natural by-product of image acquisition (remember, sampling is not perfect and noise is introduced!)
  - Many applications
    - Medical imaging
    - Industrial inspection
    - Autonomous guidance of military/space systems

Introduction (2):

- **General Idea of Sharpening Filters**
  - Recall image blurring accomplished by averaging and averaging is analogous to integration
  - Since sharpening is essentially the opposite of blurring, we can sharpen by spatial differentiation
  - We will be concerned with sharpening using digital differentiation
    - Enhances edges and other discontinuities (e.g., noise) and deemphasizes areas with slowly varying gray levels

Foundation (1):

- **Properties of Digital Differentiation**
  - Particularly interested in behavior of derivatives in areas of constant gray level (flat segments) , at the onset and end of discontinuities (step and ramp discontinuities) and along gray level ramps
    - These types of discontinuities can be used to model noise points, lines and edges in an image
  - Recall definition of derivative from calculus → think of it as a "rate of change"
    - Digital derivative similar → defined in terms "differences"

Foundation (2):

- **Properties of Digital Differentiation (cont…)**
  - Remember, we are dealing with digital values which are finite hence the largest possible gray level change (digital derivative) must also be finite and the shortest distance in which this change can occur is between two adjacent pixels

Foundation (3):

- **Defining a First Order Digital Derivative**
  - Recall digital derivative defined in terms of "differences" but many ways to define these differences
  - Any definition used to define these differences and hence a first digital derivative must conform to
    1. Must be equal to zero in flat segments
    2. Must be non-zero at the onset of a gray level step or ramp
    3. Must be non-zero along ramps

Foundation (4):

- **Defining a Second Order Digital Derivative**
  - Many ways to define a second order derivative and as with first, based on differences but all must conform to
    1. Must be zero in flat areas
    2. Must be non-zero at the onset of and end of gray level step or ramp
    3. Must be zero along ramps of constant slope