ELIC 629
Digital Image Processing

Fall 2005

Image Enhancement in the Spatial Domain:

Sharpening Spatial Filters, Introduction to edges, First order derivatives, second order derivatives

Bill Kapralos

Monday, October 31 2005

Overview (1):

- **Before We Begin**
  - Administrative details
  - Review → some questions to consider
- **Image Edges**
  - Introduction
  - Importance of edge detection
  - Modeling an edge

Overview (2):

- **Introduction to Sharpening Filters**
  - Foundation (quick review from last week)
  - First order derivatives – the gradient
  - Second order derivative

Before We Begin

Administrative Details (1):

- **No Lab Today**
  - Review during the lab period
    - In this room if there is no other class after us otherwise, in the lab
    - Optional → although it is recommended to attend, you do not have to attend
  - Lab 5 report due the day of the test

Administrative Details (2):

- **Mid-Term Exam**
  - November 7 2005
  - 6:05pm - ??? (won't be the entire period but you will have enough time) → be here on time!

Some Questions to Consider (1):

- What is spatial filtering ?
- What is a smoothing spatial filter ?
- What is an averaging filter ?
- What is a weighted averaging filter ?
- What is a sharpening filter ?
- What is a digital first derivate (define it) ?
- What criteria must be satisfied by a first order derivative ?
- What is a digital second derivate (define it) ?
- What criteria must be satisfied by a second order derivative ?

Image Edges

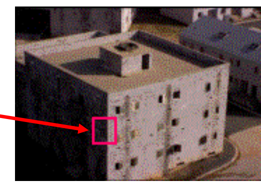
Introduction (1):

- **What is an Edge ?**
  - Intuitively → a border between two regions, where each region has (approximately) uniform brightness (gray level)
  - In an image edges typically arise from
    1. Occluding contours in an image
       - Two image regions correspond to two different surfaces
    2. Abrupt changes in surface orientation
    3. Discontinuities in surface reflectance

Introduction (2):

- **What is an Edge ? (cont…)**

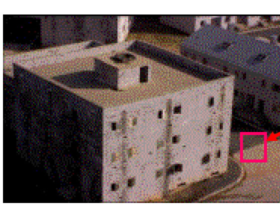
Edge due to an occluding contour



Edge due to an abrupt change in surface orientation

Introduction (3):

- **What is an Edge ? (cont…)**

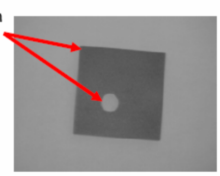


Edge due to a change in surface reflectance

Importance of Edge Detection (1):

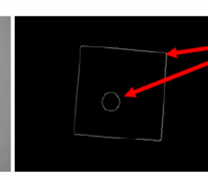
- **In Typical Images, Edges Characterize Object Boundaries/Borders**
  - Allows us to locate/identify objects in a scene e.g., to segment an image → many, many applications!

Two objects in image



Original Image

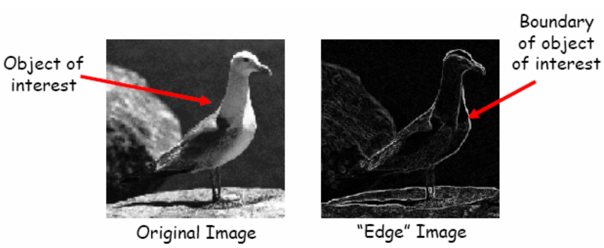
Boundary of objects in image



"Edge" Image

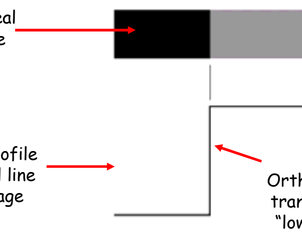
Importance of Edge Detection (2):

- **A More realistic Example**



Modeling an Edge (1):

- **Ideal Edge Model**
  - A set of connected pixels, each of which is located at an orthogonal step transition in gray level

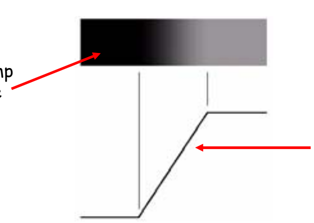


Modeling an Edge (2):

- **In Practice, Ideal Edges Don't Exist!**
  - Sampling and the fact that sampling acquisition equipment etc. is far from perfect leads to edges that are blurred
  - Changing illumination (lighting conditions) will cause changes to edges & all parts of an image in general
    - Changing lighting conditions are actually a HUGE problem for vision/image processing tasks → many algorithms will not generalize across different lighting conditions
    - Color constancy → a big field in computer vision but still an un-solved problem!

Modeling an Edge (3):

- **Reality → Edges Have a "Ramp-Like" Profile**
  - The slope of the ramp is inversely proportional to the degree of blurring in the edge
  - Updated definition → region of image in which the gray-level changes significantly over short distance



Modeling an Edge (4):

- **In Practice, Ideal Edges Dont Exist! (cont.)**
  - Edge is no longer a one-pixel thick path
    - An edge point is now any point contained in the ramp and an edge would be a set of such points which are connected
    - Thickness of edge depends on length of ramp which is determined by its slope which itself is determined by the amount of blurring
    - Blurred edges are typically thicker e.g., the greater the blurring → the thicker the edge

Sharpening Filters (Review)

Foundation (5):

- **First Order Derivative in Greater Detail**
  - Basic definition of a first order 1D function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x) \qquad \frac{\partial f}{\partial y} = f(y+1) - f(y)$$

  - Remember → above definition is of one variable (x) only since images are a function of two variables x,y e.g., f(x,y) we will be dealing with derivatives along both spatial axis "separately" hence the use of "partial derivative"

Foundation (6):

- **Second Order Derivative in Greater Detail**
  - Basic definition of a first order 1D function f(x) is the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

$$\frac{\partial^2 f}{\partial y^2} = f(y+1) + f(y-1) - 2f(y)$$

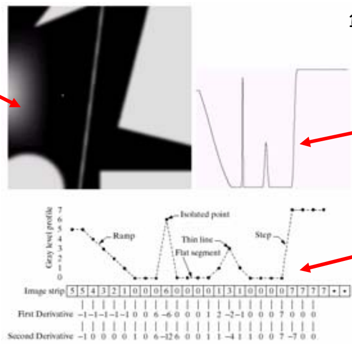
  - Once again, remember, above definition is for one variable only whereas in digital images we are dealing with two variables, x,y

Foundation (7):

- **Graphical Illustration of Digital Derivatives**

"Simple" original image

1D horizontal gray level profiles along center of image including isolated noise point



"Simplified" profile

Foundation (8):

- **Graphical Illustration Explained**
  - Traversing profile from left to right
    - First order derivative is non-zero along entire ramp but second order derivative is non-zero only at onset and end of ramp
    - Since edges in image have similar profile, we can conclude first order derivative produces "thick" edges while second order derivatives produces "finer" edges
    - A second order derivative enhances much more finer detail than first order derivative (but also enhances noise as well!)

Foundation (9):

- **Graphical Illustration Explained (cont…)**
  - To summarize
    1. First order derivatives generate thicker edges in an image
    2. Second order derivatives have stronger response to fine detail e.g., thin lines and isolated points (noise as well)
    3. First order derivatives have stronger response to gray level step
    4. Second order derivatives produce double response at step changes in gray level

Foundation (10):

- **Graphical Illustration Explained (cont…)**
  - To summarize (cont…)
    - Generally, second order derivatives are better for image enhancement as opposed to first order derivatives since they are able to enhance such fine detail

First Order Derivatives
The Gradient

Introduction (1):

- **Gradient Defined**
  - Gradient is a measure of change in a function
  - An image can be considered to be an "array" of samples of some continuous function of intensity
    - Significant changes in gray levels in image can thus be detected using discrete approximation of gradient
    - Edge detection → detecting significant local changes in an image
  - Two-dimensional equivalent of the first derivative

Introduction (2):

- **Gradient Defined (cont…)**
  - For function f(x,y) gradient of f at coordinates (x,y) is defined as a two-dimensional column vector

$$G[f(x,y)] = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

Gradient Properties (1):

- **Two Important Properties of Gradient**
  1. Vector G[f(x,y)] points in direction of maximum increase of function f(x,y)
  2. Magnitude of gradient equals maximum rate of increase of f(x,y) per unit distance in direction G. Magnitude given as

$$mag(G[f(x,y)]) = \sqrt{G_x^2 + G_y^2} = \sqrt{\left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]}$$

$$\theta = \tan^{-1}(y/x)$$

Gradient Properties (2):

- **Properties of the Gradient**
  - Components of gradient vector are linear
  - Magnitude of gradient vector is not linear given squaring and square root operations
  - Partial derivates of gradient vector are not isotropic (e.g., not rotation invariant)
  - Magnitude of gradient is isotropic
  - Often, although incorrect, we refer to the magnitude of the gradient as the gradient itself

Gradient Properties (3):

- **Properties of the Gradient (cont…)**
  - Implementing the gradient magnitude equation for an entire image is very computationally expensive and certainly not a trivial matter!
    - Approximate gradient mag. using absolute values

$$mag(G[f(x,y)]) = |G_x| + |G_y|$$

    - Above equation is easier to compute and preserves relative changes in gray levels
    - Isotropic property generally lost → as with Laplacian preserved for limited number of rotational increments, depending on mask

Approximating the Gradient (1):

- **Digital Approximation to Gradient**
  - Recall → derivatives in images are approximated by differences between pixel intensity (gray levels)
    - Gradient approximated by differences
  - For simplification, will use previous definition of 3x3 image region, where center pixel is "pixel of interest"

Sub-image region →

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Recall, z_5 denotes f(x,y), z_1 denotes f(x-1,y-1), etc.

Approximating the Gradient (2):

- **Digital Approximation to Gradient (cont…)**
  - Simplest approximation to first order derivative satisfying previously stated conditions is

$$G_x = (z_8 - z_5) \text{ and } G_y = (z_6 - z_5)$$

  - Other definitions available including one proposed by Roberts in 1965, uses "cross differences" and known as the Roberts cross gradient operators

$$G_x = (z_9 - z_5) \text{ and } G_y = (z_8 - z_6)$$

Gradient Approximations (1):

- **Roberts Cross Gradient Operator**
  - Implemented with the following masks

G_x

−1	0
0	1

G_y

0	−1
1	0

  - Difficult to implement given its "awkward" size
    - Minimum mask we are interested in is 3x3!
    - Approximation using a 3x3 mask can be given

$$G[f\{x,y)] = |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

Gradient Approximations (2):

- **Roberts Cross Gradient Operator (cont…)**
  - Implemented with the following masks

G_x

−1	−2	−1
0	0	0
1	2	1

G_y

−1	0	1
−2	0	2
−1	0	1

These two masks are known as Sobel operators

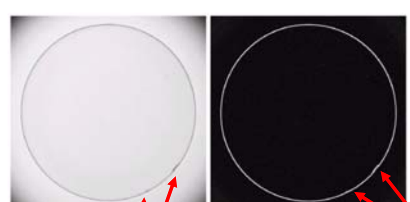
  - Difference between third and first rows approximates derivative in x direction
  - Difference between third and first columns approximates derivative in y direction

Gradient Applications (1):

- ***Many Applications and Uses***
  - Industrial applications
    - Aid humans in detecting defects → enhances defects and eliminates slowly changing background features
    - Pre-processing step in automated inspection
  - Edge detection
  - Highlight small specs not visible in gray scale image
  - Enhance small discontinuities in flat gray field

Gradient Applications (2):

- **Example of the Gradient Operator**



Optical image of contact lens with defects at the boundary

Image processed with Sobel operator → edges revealed, background eliminated and defects are more visible now