COSC 4111 3.0—Fall 2002

Posted: Sep 28, 2002
Due: End of October [Exact date TBA]

Problem Set No. 1

(1) Express each of the functions $\max(x, y)$ and $\min(x, y)$ by *substitution* using only the functions $x + y$ and $x \dot{-} y$.

(2) Prove that the function $f(z)$ that returns the number of distinct primes in the factorization of z is in $\mathcal{PR}$.

(3) Prove that $\lambda xy.\mathrm{lcm}(x, y)$ is in $\mathcal{PR}$, where "lcm" denotes *least common multiple*, i.e., the smallest number z that is divided by both x and y.

(4) Page 81, problems 18, 22.

(5) Regarding the function $\lambda ix.g_i(x)$ of Theorem 3 (p. 78–79 of text): It is proved there that $1 \dot{-} g_x(x) = 0$ is not in $\mathcal{PR}_*$.
How about $1 + g_x(x) = 0$? **Why?**

(6) Write a loop program which computes $\lambda x.\mathrm{rem}(x, 3)$. The program must only allow instruction-types $X = 0$, $X = X + 1$, $X = Y$ and **Loop** $X \ldots$ **end**. It must *not* nest the Loop-end instruction!

(7) Write a loop program which computes $\lambda xyz.$**if** $x = 0$ **then** y **else** z. The program must only allow instruction-types $X = 0$, $X = X + 1$, $X = Y$ and **Loop** $X \ldots$ **end**.

(8) Prove that the predicate $Q(z)$ that is true iff $z = 2^x + 2^y$ where $x > y > 0$ for appropriate x and y is in $\mathcal{PR}_*$.