Problem Set No. 3

Dept. of Computer Science

Date: Nov 22, 2001
Due: TBA (around Dec. 17)

1. Do Ch. 7 problems #1, 9.

2. Without using Rice's theorem, show that the set $A = \{x : \text{dom}(\phi_x) \text{ has exactly two elements}\}$ is not recursive. (I.e., "$x \in A$ is unsolvable").

3. Prove that $y = f(\vec{x})$ is r.e. iff $f \in \mathcal{P}$.

4. Do Ch. 8 problems #2, 3, 4.

NB. In #2 there's a typo: The superscript (m) should be $(m+1)$ throughout.

For #4 you may proceed with an easier proof (without selection theorem) than the one the hint on p.321 implies. Just use the previous problem.

5. Is the "proof" below correct?
"Let $y = f(\vec{x}_n)$ be r.e. Then $g = \lambda\vec{x}_n.(\mu y)(y = f(\vec{x}_n))$ is in $\mathcal{P}$. But $g = f$. Thus, $f \in \mathcal{P}$."

6. Is $\lambda x.(\widetilde{\mu} y)(y \in W_x)$ in $\mathcal{P}$? How about $\lambda x.(\mu y)(y \in W_x)$? What does the latter return?

7. (**Grad**) Show that $\{x : \phi_x(x) \downarrow\}$ is *not* a complete index set.

Hint. Start by showing (using the recursion theorem) that there is an $e \in \mathbb{N}$ such that

$$\phi_e(x) = \begin{cases} 0 & \text{if } x = e \\ \uparrow & \text{otherwise} \end{cases}$$

8. Do Ch. 9 problems #18, 20, 22.