## COSC 6113 3.0—Fall 2002

Posted: Sep 28, 2002
Due: End of October [Exact date TBA]

## Problem Set No. 1

(1) Express each of the functions $\max (x, y)$ and $\min (x, y)$ by substitution using only the functions $x+y$ and $x-y$.
(2) Find in closed form the pairing function $\lambda x y . J(x, y)$ (and its projections $\lambda z . K(z), \lambda z . L(z))$ that enumerates pairs $(x, y)$ by increasing $\max (x, y)$, and in each group with $\max (x, y)=a$ it enumerates in the order

$$
(0, a),(1, a), \ldots,(a-1, a),(a, a),(a, 0),(a, 1), \ldots,(a, a-1)
$$

(3) Write a very complete proof of $\# 25$ on p.82. You may of course consult the reference.
(4) Do $\# 32, \# 33$, p. 82.
(5) Do $\# 36, \# 37$, p.82. For $\# 36$ let me first remove an ambiguity: Clause (c) on p. 82 must be " $\langle 0, n, m, 2\rangle \in \widetilde{\mathscr{F}}$ iff $1 \leq m \leq n$ (it stands for $U 1^{n} ; 1^{m}$ )".
Hint. You may want to peek into the web notes (Normal Form Theorems) to get a feel for how "coding-decoding" is handled in the context of the inductive definition of function sets.

