COSC 2001(A and B) 3.0—Fall 2001

Date: Oct 27, 2001
Due: Nov 20, 2001

Problem Set No. 2

Papers ***must*** **be typed or word-processed (the "*must*" does not apply to diagrams)**, and deposited in a course drop-box on the due date.

► **Due time:** Any time on November 20, 2001. **Boxes will be cleared the following morning. Location of the drop-box:** There is **one** box—labeled 2001A and B—on the first floor of CCB, in the corridor that leads to the Ariel Lab.◄

In this Problem Set it is allowed—but not required!—to submit **ONE joint paper that has a total of TWO co-authors from the same section**. The same mark, as assigned to such a joint paper, will be given to **each** of its two authors.

► IFF you are submitting Problem Set #2 ***with*** a partner, then you ***must*** notify us as described below, **Prtnr1.–Prtnr4.:**

Prtnr1. Make a file called "partner" (no quotes). [Please do ***not*** call it "Partner" or "PARTNER" or "a2partner" or anything other than "partner"].

Prtnr2. Put in it your name and "prism" login, ***and*** the name and prism login of your partner as well.

Prtnr3. Give the following command on prism

"**submit 2001 a2 partner**"

NOT later than 5:00pm, Nov. 10, 2001s.

Prtnr4. Only ***one*** submission (**Prtnr3.**, above) ***per pair*** please! ◄

If you do NOT plan to work with a partner please do NOT submit any co-author information!

(1) This teamwork is **strictly for "declared" pairs**, and strictly for Problem Set #2. Teamwork may not be allowed on later assignments.

(2) Any strong similarity between different papers will be seriously frowned upon. (To learn more about this issue please follow the link "**Senate Policies**" found on the URL: http://www.cs.yorku.ca/~gt/courses/)

General Remark. Each solution must contain *adequate explanation(s)* of *why* it answers the relevant question. While examples can help us understand your point of view, *they are NOT substitutes* for a logical argument that establishes your solution's validity ***in general***.

From the text:

1. 4.2.6 (b, c), p.147

2. 4.3.5, p.154

3. 5.1.3, p.180

4. 5.2.1, p.191

5. 5.2.2, p.191

6. 6.2.2 (b), p.236

7. 6.3.2, p.245

8. 6.3.5 (c), p.246

9. **The MATH 1090 Connection.** In the MATH 1090 text (Gries and Schneider) it is described, *but not formally defined*, what the syntax of Predicate Calculus formulas is.

We would like you to define a CFG whose language is the set of all the formulas of Predicate Calculus for Arithmetic.

As you recall, each application of Predicate Calculus to a specific branch of Mathematics, like Arithmetic over the natural numbers $\mathbb{N} = \{0, 1, 2, \ldots\}$, has the need of *special symbols*—which most logicians call "nonlogical symbols".

In our case these special symbols are:

(a) 0. **Comment.** A constant symbol. If interpreted, this stands for the number 0.

(b) S. **Comment.** A function symbol of *arity* 1.[†] If interpreted, $S(x)$ stands for $x+1$. Using S and 0 we can denote all the remaining natural numbers, so we do not need any other constant symbols beyond 0 in the alphabet. E.g., $S(0)$ denotes "1", $S(S(0))$ denotes "2", etc.

(c) $+$. **Comment.** A function symbol of *arity* 2. If interpreted, $x + y$ stands for $x + y$.

(d) $\times$. **Comment.** A function symbol of *arity* 2. If interpreted, $x \times y$ stands for $x \times y$.

(e) $<$. **Comment.** A predicate symbol of *arity* 2. If interpreted, $x < y$ stands for $x < y$.

[†] "Arity" is the number of required arguments for the function or predicate symbol.

We also have the usual *required* symbols for logic (so-called *logical symbols*), namely "=" (equals; *not* to be confused with "≡" which we leave out), "∃" (we leave "∀" out), "¬", "∨", brackets—that is, "(" and ")"—and variables: of Boolean type, $p_1, p_2, p_3, \ldots$ and of integer (natural number) type, $v_1, v_2, v_3, \ldots$ We also have the two Boolean constants, denoted by "*false*" and "*true*" in the text by Gries and Schneider. Here we will denote them by the "one character" symbols "$\bot$" and "$\top$" respectively.

The task is to define a CFG (in BNF) with *start symbol* called ⟨wff⟩ such that

$$\langle\text{wff}\rangle \Rightarrow^* x \text{ iff } x \text{ is some Predicate Calculus formula (for Arithmetic)}$$

Your grammar will need other nonterminals as well, at the very least, ⟨term⟩, ⟨Bvar⟩ and ⟨Nvar⟩ so that

$$\langle\text{term}\rangle \Rightarrow^* x \text{ iff } x \text{ is some Predicate Calculus term}^\dagger$$

$$\langle\text{Bvar}\rangle \Rightarrow^* x \text{ iff } x \text{ is some Predicate Calculus Boolean variable}, p_i$$

and

$$\langle\text{Nvar}\rangle \Rightarrow^* x \text{ iff } x \text{ is some Predicate Calculus natural number variable}, v_i$$

The latter two take care of the requirement that the alphabet is finite. Thus, p_i is **really** the string

$$p\underbrace{|\ldots|}_{i \text{ of } |}p$$

and v_i is **really** the string

$$v\underbrace{|\ldots|}_{i \text{ of } |}v$$

To sum up, here is the alphabet $\mathcal{A}$ of *terminal symbols*:

$$\mathcal{A} = \{\bot, \top, v, p, |, (,), =, \exists, \neg, \vee, 0, S, +, \times, <\}$$

The nonterminal alphabet is *partly*: $\mathcal{V} = \{\langle\text{wff}\rangle, \langle\text{term}\rangle, \langle\text{Bvar}\rangle, \langle\text{Nvar}\rangle\}$.

Your task: Complete $\mathcal{V}$—if needed—and give the productions *so that the grammar is unambiguous. Explain clearly (not "by example") why your grammar IS unambiguous.*

Caution. Please do not say "it is unambiguous because every string in the language has a unique leftmost derivation". This just states the definition of "unambiguous". The question is, *why* do you think that *your grammar* is such that "every string in the language has a unique leftmost derivation"?

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† Recall that, recursively speaking, a "term" is a variable or a constant, or is an application of a function symbol on the correct number of term-arguments.