COSC 3431.03

S 1999

Date: June 22, 1999
Due: July 12, 1999

Please note: The Final Take-Home Exam will be handed out in class, on Tuesday, July 12th, 1999. **Don't miss it!**

It will be due (**in class, by 7:15pm**) on July 19th, 1999 (there will be absolutely no extensions).

Problem Set No. 3

1. Prove that the grammar in Sipser (Exercise 2.1, p. 119) produces the same language as the grammar $E \to E + E|E \times E|(E)|a$.

2. For the same grammar G above—and without converting to CNF—derive a **one state ES-PDA** that recognizes $L(G)$. Justify why your construction works.

The PDA-conventions given in class apply.

3. We can easily see that *all* possible DFA with tape alphabet $\{0, 1\}$ can be coded as strings over a fixed alphabet. Indeed, each "move" $\delta(q_i, a) = q_j$ is represented by the string $q\tilde{i} * a * q\tilde{j}$, where $a \in \{0, 1\}$ and $\tilde{i}$ is the decimal representation of the number i. Thus, using also ";" as a new symbol we represent the automaton by "gluing" the move-representations one after the other, using ";" as glue, and appending at the end the sequence $; q\tilde{m}; q\tilde{n}; \ldots ; q\tilde{k};$ indicating that q_m is the *initial* state and $q_n, \ldots , q_k$ are the *final* states.

Each automaton M has more than one string representation (due to permutations of states and/or moves being possible) over the alphabet

$$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, q, *, ;\}$$

$R(M)$ will denote all the representations of M.

Is the language $L = \big\{x; y|(\exists M)\big(M \text{ is a DFA} \quad \& \quad y \in R(M) \quad \& \quad x \in L(M)\big)\big\}$ regular? *Prove the correctness of your answer.*

4. From the text (Sipser) do #2.17(a).

5. Prove that $\{x|\text{dom}(\phi_x) = \{11, 13\}\}$ is not recursive.

6. Is the problem $\varepsilon \in L(G)$ decidable for any CFG G? How about the related problem $0 \in \text{ran}(\phi_x)$ for any number x? *In each case prove the correctness of your answer.*

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