COSC3431.03

TAKE HOME EXAM

Dept. of Computer Science

Date: July 13, 1999
Due: July 20, 1999, **In class, by 7:15pm**

1. **5 MARKS** Let NOPREFIX(L) stand for the set $\{w \in L$: no proper prefix of w is a member of $L\}$.

   Prove that if L is regular, then NOPREFIX(L) is also regular.

2. **5 MARKS** We have seen a number of "practical examples" of converting an NFA of n states to a DFA that "beat" by a substantial margin the "theoretical" upper bound (2^n) for the number of states of the DFA. This was due to the fact that many subset-states introduced were *inessential*, i.e., we could remove them *without affecting the accepted language.*

   Nevertheless, show that for any n, an NFA of n states exists whose equivalent DFA—constructed by the "subset construction"—has no less than $c2^n$ *essential* (i.e., non-removable) states, where c is a small (possibly fractional) constant.

3. **5 MARKS** Prove that the class (nondeterministic) PDAs that accept by "simultaneous empty stack and final state", ESFS-PDA, are exactly as powerful as ES-PDA or FS-PDA.

   Note that the term in quotes is meant to mean that an ESFS-PDA accepts x iff $(q_0, x, \$) \vdash^* (q, \varepsilon, \varepsilon)$ where q is *a final state.*

4. **5 MARKS** Prove that the set $\{x : \text{ran}(\phi_x)$ is infinite$\}$ is not semi-recursive.

   Hint. See how you can adapt the proof in class, that $\{x : \phi_x$ is a constant$\}$ is not semi-recursive.

5. **5 MARKS** Prove that if $P(y, \vec{x})$ is semi-recursive, then so is $(\exists y)_{<z} P(y, \vec{x})$.

6. **5 MARKS** Prove that $Q(\vec{x})$ is semi-recursive iff (note the "iff"!), for some function $f \in \mathcal{P}$ of the same number of arguments as Q, it is true that $Q(\vec{x}) \equiv f(\vec{x}) = 0$ for all $\vec{x}$.