COSC 3431.03

W. 2000

Date: Jan. 17, 2000
Due: Jan. 31, 2000—At the beginning of class, no extensions

Problem Set No. 1

General Remark. Each problem must have *adequate explanation* of why it answers the relevant question. While examples can help to understand your point of view, *they are NO substitutes* for a logical argument (this may be a "proof") that your answer is *general*, that is, it "works in all cases".

1. Start with the FA presented in Example 1.5 (p.39) of Sipser's text.

Remove the symbol "$\langle RESET \rangle$" from the alphabet (and remove the relevant parts from the automaton). Thus your alphabet now is $\Sigma = \{0, 1, 2\}$.

Prove that the so modified automaton accepts ϵ **plus** exactly those strings over Σ whose symbols (digits) have a sum divisible by 3.

Hint. There are **two** inductions needed to handle **nonempty** strings:

(a) By induction on input string length prove "if the input string has digit-sum 0 mod 3 then the computation ends at state q_0; else if it has digit-sum 1 mod 3 then the computation ends at q_1; else it ends at q_2."

(b) By induction on input string length prove "if the computation ends at state q_0 then the input string has digit-sum 0 mod 3; else if the computation ends at q_1 then the input has digit-sum 1 mod 3; else the input has digit-sum 2 mod 3."

Of course, after completing said inductions you have to explain briefly why they "solve the problem".

2. From the text (Sipser) do:

p.84, #1.4(c), #1.5(g)[**the alphabet is still** $\{0, 1\}$];

p.85, #1.10, #1.12(a), #1.13(h);

p.86, #1.16(a), #1.17(b);

3. Let for any string x over $\Sigma = \{0, 1\}$ x^R mean its reversal (i.e., x^R reads right-to-left exactly as x does left-to-right).

(a) Is $\{xwx^R : x \in \Sigma^+ \ \& \ w \in \Sigma^+\}$ regular?

Why "yes" or why "no(t)"?

(b) How about $\{xwx^R : x \in \Sigma^+ \ \& \ w \in \Sigma\}$?

Why "yes" or why "no(t)"?

Instructor: G. Tourlakis