COSC 3431.03

W. 2000

Date: March 6, 2000
Due: March 27, 2000—At the beginning of class, no extensions

Problem Set No. 3

General Remark. Each problem must have *adequate explanation* of why it answers the relevant question. While examples can help to understand your point of view, *they are NO substitutes* for a logical argument (this may be a "proof") that your answer is *general*, that is, it "works in all cases".

1. We can easily see that *all* possible DFA with tape alphabet $\{0, 1\}$ can be coded as strings over a fixed alphabet. Indeed, each "move" $\delta(q_i, a) = q_j$ is represented by the string $q\tilde{i} * a * q\tilde{j}$, where $a \in \{0, 1\}$ and $\tilde{i}$ is the decimal representation of the number i. Thus, using also ";" as a new symbol we represent the automaton by "gluing" the move-representations one after the other, using ";" as glue, and appending at the end the sequence $; q\tilde{m}; q\tilde{n}; \ldots ; q\tilde{k};$ indicating that q_m is the *initial* state and $q_n, \ldots, q_k$ are the *final* states.

Each automaton M has more than one string representation (due to permutations of states and/or moves being possible) over the alphabet

$$\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, q, *, ;\}$$

$R(M)$ will denote all the representations of M.

Is the language $L = \big\{x; y | (\exists M)\big(M \text{ is a DFA} \quad \& \quad y \in R(M) \quad \& \quad x \in L(M)\big)\big\}$ regular? *Prove the correctness of your answer.*

2. Prove that Regular languages are closed under reversal. That is, prove that if L is regular, then so is $L^R = \{x^R : x \in L\}$, where "x^R" reads from left to right *exactly* as "x" reads from right to left.

Prove this fact basing your proof on NFAs.

Caution! You may *not* use any of the following tools: Regular expressions, Regular grammars.

3. From the text (Sipser) do #2.17(a).

4. We already know that the set of Regular languages is a subset of CFL, our proof being based on Regular Grammars.

Prove this fact again, this time basing your proof on NFAs and PDAs.

Caution! You may *not* use any of the following tools: Regular expressions, grammars.

5. Prove that $\{x | \mathrm{ran}(\phi_x) = \{1, 13\}\}$ is not recursive.

6. Is the problem $\varepsilon \in L(G)$ decidable for any CFG G? How about the related problem $0 \in \mathrm{ran}(\phi_x)$ for any number x? *In each case prove the correctness of your answer.*

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