Dept. of Computer Science

AK/CS4021.03—Problem Set No. 1

Date: Sep 23, 1999
Due: TBA; *Approx.* in two weeks

(1) Show that in the presence of the *initial functions* the following operations can be simulated by composition:

(a) Substitution of a variable by a function
(b) Substitution of a variable by a constant
(c) Identification of any two variables
(d) Permutation of any two variables
(e) Introduction of new ("dummy") variables

(**NB.** In class we looked at special cases of each of (a)–(e), i.e., cases of up to 2 variables. You are to argue the general case here.)

(2) Prove that $\lambda xyz.$ **if** $x = 0$ **then** y **else** z is in $\mathcal{PR}$.

NOTE. For problem 2 *only*, please give the "fully (formally) dressed " primitive recursion, along with an informal primitive recursion. An example of the latter (for predecessor) is

$$\begin{aligned} 0 \dot{-} 1 &= 0 \\ x + 1 \dot{-} 1 &= x \end{aligned}$$

(3) Define

$$(\mathring{\mu} y)_{\leq z} f(y, \vec{x}) \overset{\text{def}}{=} \begin{cases} \min\{y : y \leq z \;\; \& \;\; f(y, \vec{x}) = 0\} \\ 0, \text{ if the min does not exist} \end{cases}$$

Prove that $\mathcal{PR}$ is closed under $(\mathring{\mu} y)_{\leq z}$.

(4) (**Tricky, but has a "one-liner proof"**) Prove that if a class of functions $\mathcal{C}$ is closed under $(\mathring{\mu} y)_{\leq z}$ *and* substitution, then its corresponding class of relations $\mathcal{C}_*$ is closed under $(\exists y)_{\leq z}$.

(5) Show that if $\lambda \vec{x} y . y = f(\vec{x})$ is in $\mathcal{PR}_*$, and f is total, *and* $f(\vec{x}) \leq g(\vec{x})$ for all $\vec{x}$, where $g \in \mathcal{PR}$, then $f \in \mathcal{PR}$.

(6) Write a loop program which computes $\lambda x . \lfloor x/4 \rfloor$. The program must only allow instruction-types $X = 0$, $X = X + 1$, $X = Y$ and **Loop** $X \ldots$ **end**. It must *not* nest the Loop-end instruction!