CS4021.03

Problem Set No. 3

Dept. of Computer Science

Date: Nov 11, 1999
Due: In last class

1. Do Ch. 7 problems #1, 4.

2. Without using Rice's theorem, show that the set $A = \{x : \text{dom}(\phi_x) \text{ has exactly two elements}\}$ is not recursive. (I.e., "$x \in A$ is unsolvable").

3. Do Ch. 8 problems #2, 3, 4.

NB. In #2 there's a typo: The superscript (m) should be $(m+1)$ throughout.

You may proceed with an easier proof (without selection theorem) of #4 than the hint on p.321 implies: Just recall that f is in $\mathcal{P}$ iff its *graph* is r.e.

4. Indeed, show that $y = f(\vec{x})$ is r.e. iff $f \in \mathcal{P}$.

Now check the following argument for the only-if part (I hope that was *not* the argument you used!) and explain what is *wrong* with it.

Argument. "Sure. Just observe that $f(\vec{x}) = (\mu y)(y = f(\vec{x}))$. Since the predicate $y = f(\vec{x})$ is r.e., applying μ to it gets us a $\mathcal{P}$-function."

5. So μ does not work above (and you hopefully gave reasons why). Show that you can use the Selection function instead (of problem 2 or 3 in chapter 8) to do what μ failed to do for you, and provide a proof (different than the one you gave in the previous problem) that if $y = f(\vec{x})$ is r.e., then $f \in \mathcal{P}$.

6. Show that $\{x : \phi_x(x) \downarrow\}$ is *not* a complete index set.

Hint. Start by showing (using the recursion theorem) that there is an $e \in \mathbb{N}$ such that

$$\phi_e(x) = \begin{cases} 0 & \text{if } x = e \\ \uparrow & \text{otherwise} \end{cases}$$