A SIMPLE INDUCTION PROOF

Prove

$$1 + 3 + 5 + \ldots + (2n + 1) = \sum_{i=0}^{n} (2i + 1) = (n + 1)^2 \tag{1}$$

Proof. Simple Induction on n. *Basis*: For $n = 0$ we have

$$1 = \sum_{i=0}^{0} (2i + 1) = 2.0 + 1 = 1^2$$

Correct!

Now fix n, and take as *I.H.* (1) above.

For the *I.S.* I consider the $n+1$ case, for the same n that I fixed above: Simple arithmetic yields

$$\sum_{i=0}^{n+1} (2i + 1) \overset{\text{I.H. see red to the right}}{=} (n + 1)^2 + 2(n + 1) + 1 = (n + 2)^2 \qquad \square$$