Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 M. Problem Set No3

Posted: Feb. 14, 2020

Due: Mar. 3, 2020; by 3:00pm, in the course assignment box.

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (5 MARKS) Prove that if $f : A \to B$ is a 1-1 correspondence, then so is f^{-1} and we have $ff^{-1} = \mathbf{1}_B$ and $f^{-1}f = \mathbf{1}_A$.

2. (5 MARKS) Let $f : A \to B$. Then $(\mathbf{1}_B f) = f$ and $(f\mathbf{1}_A) = f$.

3. (6 MARKS) Let $f : A \to B$ be a 1-1 correspondence. Then

- If $gf = \mathbf{1}_A$, we have $g = f^{-1}$.
- If $fh = \mathbf{1}_B$, we have $h = f^{-1}$.

4. (6 MARKS) Show that the range of the function $f : \mathbb{N}^2 \to \mathbb{N}$ of variables x, y given by $f(x, y) = 2^x 3^y$ has infinite range.

Hint. Prove that the set $A = \{2^x 3^0 : x \in \mathbb{N}\}$ is infinite —can you *easily* show that it is in 1-1 correspondence with a set we *know* is infinite? Conclude that so is $\text{ran}(f)$.

5. (5 MARKS) Prove that if A and B are enumerable, so is $A \times B$.