Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

EECS 1028 M. Problem Set No2

Posted: Feb. 2, 2022

Due: Feb. 17, 2022; by 3:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (4 MARKS) Let $\mathbb{F}$ be a nonempty family of transitive relations. Prove that $\bigcap \mathbb{F}$ is a transitive relation.

Hint. This requires two parts to be settled in the proof.

2. (3 MARKS) Let $A \neq \emptyset$ be a set. Prove that A^2 is an equivalence relation on A.

3. (4 MARKS) Prove that if a *reflexive* R (on some set A) satisfies

$$xRy \wedge xRz \rightarrow yRz \tag{1}$$

for all x, y, z, then it is an *equivalence relation*.

Caution. Note the order of the x, y, z in (1)!

4. (5 MARKS) Show that if for a relation R we know that $R^2 \subseteq R$, then R is transitive, *and conversely*.

Hint. There are two directions in the sought proof.

5. (5 MARKS) Without using the $\bigcup_{i=1}^{\infty} R^i$ representation of R^+,

Prove that for any set relation R (we did not pick it to be "on" any particular set)

$$R^+ = \bigcap \Big\{ Q : R \subseteq Q \wedge \ Q \text{ is transitive} \Big\}$$

Hint. You may use Questions 1 and 2 above even if you did not prove them.

6. (5 MARKS) Give an example of two equivalence relations R and S on the set $A = \{1, 2, 3\}$ such that $R \cup S$ is *not* an equivalence relation.