Lassonde School of Engineering

Dept. of EECS

Professor G. Tourlakis

EECS 1028 Z. Problem Set No2

Posted: Feb. 3, 2024

Due: Feb. 19, 2024; by 6:00pm, in eClass.

Q: **How do I submit?**

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (3 MARKS) Give an example of two equivalence relations R and S on the set $A = \{1, 2, 3\}$ such that $R \cup S$ is *not* an equivalence relation.

2. (3 MARKS) Let P be a reflexive relation on A that satisfies $aPb \wedge aPc \rightarrow bPc$. Prove that P is an equivalence relation on A.

Caution. This $aPb \wedge aPc \rightarrow bPc$ is <u>not</u> exactly transitivity!

3. (3 MARKS) Show for a relation $\mathbb{S}$ that if both the range and the domain are sets, then $\mathbb{S}$ is a set.

4. (3 MARKS) Let $A \neq \emptyset$ be a set. Prove that A^2 is an equivalence relation on A.

5. (4 MARKS) Let R be symmetric. Show that so is R^n for the arbitrary $n > 0$.

Hint. No need for induction. Show this by noting (from class that) $R^n = \overbrace{R \circ \cdots \circ R}^{n\ R}$.

6. (3 MARKS) Show that a relation $\mathbb{R}$ is symmetric iff $\mathbb{R} = \mathbb{R}^{-1}$.

Caution. There are two directions here.

7. (3 MARKS) Show that if a relation $\mathbb{S}$ is transitive, then so is $\mathbb{S}^{-1}$.

8. (5 MARKS) Let R on A be reflexive and symmetric. Prove that R^+ is an equivalence relation.