EECS 4111/5111 —Fall 2021

Posted: Nov. 5, 2021
Due: TBA by a NEWS item on the course web page.
You have a window of three weeks at least.

Problem Set No. 2

NB. *All problems are equally weighted out of 5.* The problem set list for *grad students* enrolled in EECS5111 is the entire list here. Undergrads *should omit the problems marked "***Grad only***"*.

This is not a course on *formal* recursion theory. Your proofs should be *informal* (but NOT sloppy), *completely argued*, correct, and informative (and if possible *short*). Please do not trade length for correctness or readability.

All problems are from the "Theory of Computation Text", or are improvisations I completely articulate here.

(1) Do Exercise 2.5.0.30 (p.171, *Definition by Positive Cases*).

From Section 2.12.

(2) (**Grad only**) Do problems 22, 23.

Moreover, in exercise 35 it asks you to elaborate on the comment "In general". 'Why "in general"?', it asks. Can you give an example where for two partial computable f and g the function

$$h = \begin{cases} f & \text{if } \phi_x(x)\downarrow \\ g & \text{if } \phi_x(x)\uparrow \end{cases}$$

is in $\mathcal{P}$?

(3) Do problem 26, 36, 43.

(4) Do problem 43.

Rice's theorem and relevant lemmata are not allowed in the following. Argue from scratch finding suitable reductions.

(5) Do problems 45, 46, 53.

(6) Prove that there is a $g \in \mathcal{P}$, such that $W_x \neq \emptyset$ implies $g(x) \downarrow$ and $g(x) \in W_x$.

Hint. To *define* the value "y" of $g(x)$ you want, for any given x, a y such that $y \in W_x$ —that is, $\phi_x(y) \downarrow$.

Start with the projection theorem and let Q be *recursive* such that

$$y \in W_x \equiv (\exists z)Q(z, y, x)$$

Now dovetail the verification of $(\exists z)Q(z, y, x)$ to find a y and z that, for the given x, the triple (z, y, x) makes $Q(z, y, x)$ true. Coding (z, y) as $w = \langle z, y \rangle$ —thus $y = (w)_1$— *enumerate* the w until you find one such that $Q((w)_0, (w)_1, x)$ is true. You want to return $(w)_1$ as "$g(x)$". Make all this coherent and mathematically precise!