York University

Faculties of Science and Engineering, Arts, Atkinson
MATH 1090. Problem Set #4
Posted November 20, 2005

Due: December 8, 2005; 4:00pm, in the course box

Section A

Reminder: As you know, the final exam, which will be common for sections A and B, is scheduled as follows: SC/MATH 1090 3.00 A and B; Thu, 8 Dec 2005; 19:00–22:00; CSE B

Worth reproducing (from the course outline):

"The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, at the end of all this consultation each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

See http://www.yorku.ca/secretariat/legislation/senate/acadhone.htm to familiarise yourselves with Senate's expectations regarding Academic Honesty.

The concept of late assignments does not exist."

In what follows, "prove $\vdash A$" means give a proof of A in *any* of the styles we have learnt —Hilbert, Equational, Resolution, by-Post, etc. unless a particular methodology is requested. Corresponding comment holds for "prove $\Gamma \vdash A$": Prove A from assumptions Γ.

(5 MARKS/Each) **Do the following problems**.

Important. If in any problem where you use a technique different than the one requested, then your maximum points will be 2.

Appropriate annotation is always required!

1. Use Resolution (in combination with the Deduction Theorem) —but NOT Post's theorem— to prove $\vdash (p \Rightarrow (q \Rightarrow r)) \Rightarrow (q \Rightarrow (p \Rightarrow r))$.

2. Do 9.9 (p.174) from Gries/Schneider.

You must translate to standard notation before you start your proof.

3. Do 9.16 (p.174) from Gries/Schneider.

You must translate to standard notation before you start your proof.

4. Do 9.17 (p.174) from Gries/Schneider.

You must translate to standard notation before you start your proof.

5. Do 9.18 (p.174) from Gries/Schneider.

You must translate to standard notation before you start your proof.

6. Do 9.20 (p.175) from Gries/Schneider.

You must translate to standard notation before you start your proof.

7. Do 9.25 (p.175) from Gries/Schneider.

You must translate to standard notation before you start your proof.

8. Do 9.28 (p.175) from Gries/Schneider.

You must translate to standard notation before you start your proof.

9. Do Exercise 7.2.9 from the web notes.

NB. The given hints suggest possible interpretations to solve the problem. These are not the only interpretations that work. Always choose simple interpretations that work.

10. Do Exercise 7.2.10 from the web notes.

NB. The given hints suggest possible interpretations to solve the problem. These are not the only interpretations that work. Always choose simple interpretations that work.

11. Axiom 4 says that no matter for which choice of A and x,

$$(\forall x)(A \Rightarrow B) \Rightarrow (\forall x)A \Rightarrow (\forall x)B$$

can be inserted in a proof, hence is a theorem.

Prove by an *appropriate specific choice* of A that the converse,

$$((\forall x)A \Rightarrow (\forall x)B) \Rightarrow (\forall x)(A \Rightarrow B) \tag{1}$$

is not a logically valid schema.

Conclude with reason (one sentence) that (1) cannot be a theorem schema.

NB. Always choose simple interpretations that work.

12. Notwithstanding problem #9 above, I think I have a proof that actually

$$\vdash (\forall x)(\exists y)A \Rightarrow (\exists y)(\forall x)A$$

Here it is (splitting the $\Rightarrow$ and going via the deduction theorem):

(1)	$(\forall x)(\exists y)A$	$\langle$hypothesis$\rangle$
(2)	$(\exists y)A$	$\langle$(1) + spec$\rangle$
(3)	$A[y := z]$	$\langle$assume; z is fresh$\rangle$
(4)	$(\forall x)A[y := z]$	$\langle$(3) + gen; OK because x is not free in line (1)$\rangle$
(5)	$(\exists y)(\forall x)A$	$\langle$(4) + "dual of spec" (5.5.2 in the web notes)$\rangle$

Hmm ... If you believe the result of #9, then the above proof must be wrong, right?

Precisely which step, and why, is wrong?

13. You must use the "auxiliary variable metatheorem" in this proof.

Prove $\vdash (\exists x)(A \Rightarrow B) \Rightarrow (\forall x)A \Rightarrow (\exists x)B$.

14. You must use the "auxiliary variable metatheorem" in this proof.

Prove $\vdash (\exists x)B \Rightarrow (\exists x)(A \vee B)$.