Faculty of Science and Engineering

Dept. of Mathematics and Statistics

MATH1090. Problem Set No3

Posted: Oct. 22, 2007

Due: Nov. 16, 2007; in the course assignment box.

It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.

A terse but full annotation of each proof step is required! In what follows, "prove $\vdash A$" means give a proof of A in *any* of the styles we have learnt —Hilbert, equational, resolution, by-Post, etc. unless a particular methodology is requested. Corresponding comment holds for "prove $\Gamma \vdash A$": Prove A from assumptions Γ.

Do the following problems.

Important. If in any problem where you use a technique different than the one requested, then your maximum points will be 2.
Appropriate annotation is always required!

1. (5 MARKS) Use resolution (in combination with the deduction theorem) —but NOT Post's theorem— to prove $A \to B \vdash C \vee A \to C \vee B$

2. (5 MARKS) Use resolution (in combination with the deduction theorem) —but NOT Post's theorem— to prove $A \to (B \to C),\ B \vdash A \to C$

3. (5 MARKS) Use resolution (in combination with the deduction theorem) —but NOT Post's theorem— to prove $\vdash (p \to (q \to r)) \to (q \to (p \to r))$.

4. (5 MARKS) Prove that $A \vee A \vee A \vdash\ B \to A$

5. (6 MARKS) Let Σ be an infinite set of formulae for which we know that *every finite subset* is satisfiable. Prove that the entire Σ is satisfiable.

Hint. This uses Post's theorem.

6. (2 MARKS) In the formula

$$(\forall x)\Big((\forall x)(\forall y)x < y \vee x > z\Big) \rightarrow (\forall y)y = x$$

find to which "$(\forall x)$", if any, each occurrences of x belongs. Here "$<$" and "$>$" are just some nonlogical symbols (predicates of arity 2) of the alphabet.

7. (5 MARKS) Prove —by induction on terms— that for any terms t and s, if s is a *prefix* of t, then the strings t and s *must* be identical.

8. (5 MARKS) Prove

$$\vdash (\forall \mathbf{x})(A \rightarrow B \rightarrow C) \rightarrow (\forall \mathbf{x})(A \rightarrow B) \rightarrow (\forall \mathbf{x})(A \rightarrow B)$$

9. (5 MARKS) Prove

$$\vdash A \vee (\forall \mathbf{x})_B C \equiv (\forall \mathbf{x})_B (A \vee C), \text{ as long as } \mathbf{x} \text{ not free in } A$$

Hint. Translate first to standard notation.

10. (5 MARKS) Prove

$$\vdash A \wedge (\exists \mathbf{x})_B C \equiv (\exists \mathbf{x})_B (A \wedge C), \text{ as long as } \mathbf{x} \text{ not free in } A$$

Hint. Translate first to standard notation.