Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis
MATH1090 B. Problem Set No. 4
Posted: Nov. 20, 2021

Due: Dec. 8, 2021; by 5:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

Unless a required proof style (e.g., by resolution, Equational, Hilbert) is used in your answer, then your answer is graded out of 0.

(5 POINTS Max for each question) **Do all of the following:**

All resolution proofs below MUST use the graphical technique. Minimise preprocessing. You lose marks if your preprocessing is so long that it solves the problem WITHOUT doing any resolution step.

1. Prove using soundness that the **converse** of Axiom 2 is NOT a theorem.

2. Prove using soundness that $(\exists \mathbf{x})A \to A[\mathbf{x} := \mathbf{z}]$ where $\mathbf{z}$ is fresh for $(\exists \mathbf{x})A$ is NOT a theorem.

Conclude that $(\exists \mathbf{x})A \vdash A[\mathbf{x} := \mathbf{z}]$ is not a valid statement.

3. Using the auxiliary hypothesis metatheorem prove

$$\vdash (\exists x)A \to (\exists x)\Big(A \wedge (A \vee B)\Big)$$

4. What is wrong with the following proof of

$$(\forall y)(\exists x)A \vdash (\exists x)(\forall y)A? \qquad (\dagger)$$

1) $(\forall y)(\exists x)A$ $\langle$hyp$\rangle$
2) $(\exists x)A$ $\langle 1 + \text{spec}\rangle$
3) $A[x := z]$ $\langle$auxiliary hypothesis for 1); z fresh$\rangle$
4) $(\forall y)A[x := z]$ $\langle$gen + 3; OK, no free y in "hyp" line 1)$\rangle$
4′) $\Big((\forall y)A\Big)[x := z]$ $\langle$4) and 4′) are the same by def. of $[x := z]\rangle$
5) $(\exists x)(\forall y)A$ $\langle 4' + \text{Dual spec}\rangle$

However we proved in class and NOTES that (†) cannot be proved!