This page must be submitted as the first page of your MidTerm-paper answer pages.

York University
Department of Electrical Engineering and Computer Science
Lassonde School of Engineering

MATH 1090B. MID TERM, October 21, 2021; 11:30(am)-13:00(pm)

Professor George Tourlakis

By putting my name and student ID on this MID TERM page, I attest to the fact that my answers included here and submitted by Moodle are my own work, and that I have acted with integrity, abiding by the *Senate Policy on Academic Honesty* that the instructor discussed at the beginning of the course and *linked the full Policy to the Course Outline*.

Student NAME (Clearly): _____

Student NUMBER (Clearly): _____

DATE (Cearly): _____

README FIRST! INSTRUCTIONS:

1. Please read ALL these instructions carefully before you start writing.
2. This is a **TIME-LIMITED ON LINE *MID TERM***. You have **90 MIN to answer the MidTerm questions**. ABSOLUTELY last opportunity to upload is **BY 13:15 (pm)** —that is 15min MAX allocated to upload your answers to eClass. **Only ONE file can be uploaded per student**.
3. If you submit photographed copy **it still must be ONE file that you submit**. Either ZIP the PNG or JPEG images **OR** import them in MS Word and submit *ONE* Word *file* with the photos attached.
4. Using the time allotted for the uploading mechanisms (15 min) for the MidTerm-**answering** part is at your own *risk*. **MidTerm not uploaded on time = MidTerm not written**.
5. Please write your answers by hand —see also 3. above— **as you normally do for assignments** or use a word processor that can convert to PDF. MS Word is acceptable to upload as is (without conversion to PDF).
6. Whichever theorems were *proved* in class or appeared in the assignments you may use without proof, **unless I am asking you to prove them in this MidTerm**. If you are not sure whether some statement has **indeed** been proved *in class*, I recommend that you prove it in order to be "safe".

Question	MAX POINTS	MARK
1	3	
2	6	
3	5	
4	5	
5	5	
6	8	
TOTAL	32	

The following list presents and names the logical axiom schemata (note that brackets are selectively used!). A, B, C name arbitrary formulae (thus are meta variables).

Properties of $\equiv$

Associativity of $\equiv$ $((A \equiv B) \equiv C) \equiv (A \equiv (B \equiv C))$ (1)

Symmetry (commutativity) of $\equiv$ $(A \equiv B) \equiv (B \equiv A)$ (2)

Properties of $\bot, \top$

$\top$ and $\bot$ $\top \equiv \bot \equiv \bot$ (3)

Properties of $\neg$

Introduction of $\neg$ $\neg A \equiv A \equiv \bot$ (4)

Properties of $\vee$

Associativity of $\vee$ $(A \vee B) \vee C \equiv A \vee (B \vee C)$ (5)

Symmetry of $\vee$ $A \vee B \equiv B \vee A$ (6)

Idempotency of $\vee$ $A \vee A \equiv A$ (7)

Distributivity of $\vee$ **over** $\equiv$ $A \vee (B \equiv C) \equiv A \vee B \equiv A \vee C$ (8)

Excluded Middle $A \vee \neg A$ (9)

Properties of $\wedge$

"Golden Rule" $A \wedge B \equiv A \equiv B \equiv A \vee B$ (10)

Properties of $\rightarrow$

Implication $A \rightarrow B \equiv A \vee B \equiv B$ (11)

Primary rules of Inference:

$$\frac{A, A \equiv B}{B} \quad \text{Eqn}$$

$$\frac{A \equiv B}{C[\mathbf{p} := A] \equiv C[\mathbf{p} := B]} \quad \text{Leibniz}$$

Question 1. (3 MARKS) *Syntactic proofs will NOT be accepted in this question*!

Through truth tables or related short cuts show that, for any formulas A and B, we have

$$A \models_{\text{taut}} B \text{ iff } \models_{\text{taut}} A \to B$$

Question 2. Prove that the following are ***not*** wff.

The proof in each case must be by analysing either formula constructions, or the recursive definition of formulas.

(a) (4 MARKS) $(\mathbf{p}\neg\mathbf{q})$

(b) (2 MARKS) $(())$

Question 3. (5 MARKS) Give an **Equational** proof of $\bot \vdash \top$.

Limitations to allowed tools: *May **NOT*** use:

- Post's Theorem
- Deduction Theorem
- Resolution
- Cut Rule
- Hilbert style proof

Use of any of the listed above tools will entail that 0 MARKS will be earned in this problem.

Question 4. (5 MARKS) Give an **Equational** proof. Any other proof **maxes** to **zero**.

"If $\vdash A$ and $\vdash B$, then $\vdash A \equiv B$"

Question 5. (5 MARKS) *Prove* via an **Equational proof** the following:

$$\vdash A \vee A \wedge B \equiv A$$

Any other proof maxes to zero!

Question 6. For any A and B *prove* each of the following. *Equational proofs **must*** be used **in each** of (a) and (b), and you may **NOT** use Post's Theorem or the Deduction Theorem!

(a) (4 MARKS) $A \wedge B \vdash A \equiv B$

(b) (4 MARKS) $B \wedge \neg B \vdash \bot$