MATH 1090.03

Winter 2000

Date: Feb. 3, 2000
Due: Feb. 22, 2000; **At the beginning of class.**

Problem Set No. 3

The midterm test is on Tuesday, Feb. 22, 2000, duration 1h 15min, starting at 10:00am, ending at 11:15am.

The test will be *closed book*, but I will provide you with the list of the 10 or so logical axioms (schemata) from Gries and Schneider (however, you must *remember* the three "primary" rules, *equanimity, Leibniz, and transitivity*—as well as derived rules such as *modus ponens*, *cut-rule*, as well as the *Deduction Theorem* and *Proof by contradiction* metatheorems.)

- Do the following problems from the text, Chapter 3.

  3.34, 3.37, 3.55, 3.56, 3.58, 3.61–3.64, 3.67.

Hint for the above. Ignore the hints!

- Using the Deduction Theorem and the "cut rule" as your central tools—but do *not* use the "Tautology Theorem"—prove (A, B, C are any given wffs):
  (1) $\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$
  (2) $\vdash (A \Rightarrow C) \Rightarrow (B \Rightarrow C) \Rightarrow (A \vee B \Rightarrow C)$.
- In this exercise you are allowed to use Post's Tautology Theorem (and any other "valid" tools we have "built" in class).

Prove for any A, B, C (wffs) that
  (1) $\vdash A \wedge B \Rightarrow C \equiv A \Rightarrow B \Rightarrow C$
  (2) Also do, Ch. 4, p.80, 4.2, 4.5, 4.12.

- Give a "Hilbert-style proof" that *if all we know is*:
  Nothing about logical axioms, but do have at our disposal *just the two rules*
  (i) "Equanimity-1":
  $$A, A \equiv B \vdash B$$
  and
  (ii) "Leibniz":
  $$A \equiv B \vdash C[p := A] \equiv C[p := B]$$
  then we can (meta)*prove* the rule "transitivity". That is, it is a *derived rule*!
  $$A \equiv B, B \equiv C \vdash A \equiv C.$$