

The logical axiom schemata used in class are given here for your reference:

These axiom schemata (1)–(12) are “good” just for those questions that you are asked to do with “Chapter 3 Techniques”.

In your answers please refer to them by name or by the numbers given here (*not by numbers in the text!*).

Associativity of \equiv	$((A \equiv B) \equiv C) \equiv (A \equiv (B \equiv C))$	(1)
Symmetry of \equiv	$(A \equiv B) \equiv (B \equiv A)$	(2)
<i>true</i> : Identity of \equiv	$true \equiv A \equiv A$	(3)
<i>false</i> : Negation of \equiv	$false \equiv \neg true$	(4)
	$\neg(A \equiv B) \equiv \neg A \equiv B$	(5)
Associativity of \vee	$(A \vee B) \vee C \equiv A \vee (B \vee C)$	(6)
Symmetry of \vee	$A \vee B \equiv B \vee A$	(7)
Idempotency of \vee	$A \vee A \equiv A$	(8)
Distributivity of \vee over \equiv	$A \vee (B \equiv C) \equiv A \vee B \equiv A \vee C$	(9)
Excluded Middle	$A \vee \neg A$	(10)
Golden Rule	$A \wedge B \equiv A \equiv B \equiv A \vee B$	(11)
Implication	$A \Rightarrow B \equiv A \vee B \equiv B$	(12)

The following are the axiom schemata for Ch.9 and 8: *The partial generalizations of each formula in groups **Ax1**–**Ax6**.*

Ax1. All formulas in **Taut**.

Ax2. For every formula A , $(\forall x)A \Rightarrow A[x := t]$, for any term t . Recall that *the notation implies* no capture substitution. We say that “ t must be *substitutable in* x ”.

Ax3. For every formula A and variable x *not free in* A , $A \Rightarrow (\forall x)A$.

Ax4. For every formulas A and B , $(\forall x)(A \Rightarrow B) \Rightarrow (\forall x)A \Rightarrow (\forall x)B$.

Ax5. For *each* object variable x , the formula $x \approx x$.

Ax6. (*Leibniz’s characterization of equality—1st order version.*) For any formula A , any object variable x and any term t , the formula $x \approx t \Rightarrow (A \equiv A[x := t])$.

Primary rules of inference are **Equanimity** and **PSL** in both Ch.3 and Ch.9. We know that SLCS and WLUS (as well as transitivity) are **derived rules**. We know that (weak) generalization is a derived rule, namely, “If $\Gamma \vdash A$ so that any formulas from Γ actually used in the proof had **no free x occurrences**, then also $\Gamma \vdash (\forall x)A$.”

I finally note the following “general” monotonicity rule for your convenience: If $\Gamma \vdash A \Rightarrow B$ so that any formulas from Γ actually used in the proof had **no free x occurrences**, then we can infer both

- (1) $\Gamma \vdash (\forall x)A \Rightarrow (\forall x)B$, and
- (2) $\Gamma \vdash (\exists x)A \Rightarrow (\exists x)B$