York University

Faculties of Pure and Applied Science, Arts, Atkinson

MATH 2090. Problem Set #1. Posted January 19, 2002

Due in the Tutorial of Friday, January 25, 2002

Section N

In your proofs it is imperative to clearly state what **tools** you use (e.g., WLUS, SLCS, MP, PSL, Monotonicity, Deduction Theorem, Weak Generalization, which axiom(s), etc.)

1. (5 MARKS) In class we have proved that $\vdash (\exists x)(\forall y)A \Rightarrow (\forall y)(\exists x)A$ using the Auxiliary Variable Metatheorem.

Now prove the same thing but without the help of the Auxiliary Variable Metatheorem.

2. (5 MARKS) Prove that $\vdash ((\forall x)B \Rightarrow A) \equiv (\exists x)(B \Rightarrow A)$, *provided x is not free in A.*

Use an equational proof!

3. (5 MARKS) Let f be a function symbol of arity 1. Prove that $\vdash x \approx y \Rightarrow fx \approx fy$.

Careful! Do *not* just say that this "follows" from Chapter 1 stuff. It doesn't. This is a Predicate Calculus exercise!

4. (5 MARKS) Prove the metatheorem "$\forall$ commutativity (symmetry)" below.

$$\vdash (\forall x)(\forall y)A \equiv (\forall y)(\forall x)A$$

5. Let P be **any predicate of arity 2**.

(a) (2 MARKS) Explain WHY $(\forall x)(\forall y)P(x,y) \Rightarrow (\forall y)P(y,y)$ is NOT an instance of **Ax2**.

(b) (5 MARKS) Nevertheless, *prove* that $(\forall x)(\forall y)P(x,y) \Rightarrow (\forall y)P(y,y)$ IS an absolute theorem.

6. (5 MARKS) Prove (absolutely) the formula $(\exists x)A \wedge (\forall x)B \Rightarrow (\exists x)(A \wedge B)$.

7. (5 MARKS) Prove (absolutely) the formula $(\exists x)(A \Rightarrow (\forall x)A)$

A *may* have free x occurrences!

8. (5 MARKS) Let x, y, z be distinct variables and P, Q predicates of one variable.

Prove the formula

$$Q(x) \Rightarrow (\forall y)\Big(Q(y) \Rightarrow (\forall z)P(z)\Big) \Rightarrow (\forall x)P(x)$$

9. (5 MARKS) For any predicate P of arity 2 prove

$$\vdash (\forall x)(\forall y)P(x,y) \equiv (\forall y)(\forall x)P(y,x)$$

This is different from problem #4. (How?)