York University

Faculties of Pure and Applied Science, Arts, Atkinson
MATH 2090. Problem Set #3. Posted February 8, 2002

Due in the Course Box. Monday, February 25, 2002

Section N

In your proofs it is imperative to clearly state what **tools** you use (e.g., WLUS, SLCS, MP, PSL, Monotonicity, Deduction Theorem, Weak Generalization, which axiom(s), etc.)

1. Recall from class that "$(\Sigma i|0 \le i < n : i)$" means $\sum_{i<n} i$ and the latter is a function $f(n)$ of the variable n over $\mathbb{N}$ given inductively by

$$f(0) = 0$$
$$f(n+1) = f(n) + n$$

After this clarification/reminder do 12.4(b), p.242.

2. Also do from the text p.244–246:

$$\{12.9, 12.15, 12.20\}$$

In problem 12.15 attach the *intuitive* meaning of number of elements to the symbol $\#A$, where A is a set (i.e., never mind the GS "axiom 11.12").

Conventions **AND** notation from class apply!

(a) Convert GS-assertions to "standard mathematical notation" before you start any proof.

(b) *Do not* use Peano formal axioms in any of the above proofs

Also do

- Prove, this time *formally* from Peano Axioms (see Web-posting),

$$\mathrm{PA} \vdash (\forall x, y, z)(x + y) + z = x + (y + z)$$

NB. "$(\forall x, y, z)$" is a lazy way to write "$(\forall x)(\forall y)(\forall z)$".

Hint. Do (simple) induction on z.

- Prove, *formally* from Peano Axioms

$$\mathrm{PA} \vdash (\forall x) 0 + x = x$$

Hint. **Caution!** You do *not* have commutativity **yet**! Do (simple) induction on x.

- Now do simple induction on y to prove commutativity: Prove, *formally* from Peano Axioms

$$\mathrm{PA} \vdash (\forall x, y) x + y = y + x$$

Hint. Prove, *formally* from Peano Axioms, a (trivial) Lemma: $\mathrm{PA} \vdash (\forall x) Sx = x + S0$. Then use associativity and previous problem.