AK/MA2441.03. Problem Set No. 1. **Induction (Part I)**.

Date: Mon May 10, 1999
Due: May 24, 1999 (in class)

Reading material: Class notes, and Epp (2nd Edition, Chapter 4, *except* Section 4.5).

1. Using induction prove that $1^3 + 2^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$, for $n \geq 1$.

2. Using induction prove that $\sum_{i=1}^{n+1} i2^i = n2^{n+2} + 2$, for $n \geq 0$.

3. Using induction prove that $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}}$, for $n \geq 2$.

4. Let

$$\begin{aligned} b_1 &= 3,\ b_2 = 6 \\ b_k &= b_{k-1} + b_{k-2}, \text{ for } k \geq 3 \end{aligned}$$

Prove by induction that b_n is divisible by 3 for $n \geq 1$. (Be careful to distinguish between what is *basis* and what are *cases* arising from the **induction step**! As you know, our text is careless about this.)

5. Let

$$\begin{aligned} b_0 &= 1,\ b_1 = 2,\ b_3 = 3 \\ b_k &= b_{k-1} + b_{k-2} + b_{k-3}, \text{ for } k \geq 3 \end{aligned}$$

Prove by induction that $b_n \leq 3^n$ for $n \geq 0$. (Once again, be careful to distinguish between what is *basis* and what are *cases* arising from the **induction step**!)

6. Prove that

$$\sum_{0 \leq k \leq n} (-2)^k = (1/3)(1 - 2^{n+1})$$

for all *odd positive* n.

7. Prove that $2^{2n+1} + 3^{2n+1}$ is divisible by 5 for all $n \geq 0$.

8. Let

$$\begin{aligned} F_0 &= 0,\ F_1 = 1 \\ F_k &= F_{k-1} + F_{k-2}, \text{ for } k \geq 2 \end{aligned}$$

Let ϕ stand for the number $\frac{1+\sqrt{5}}{2}$.

Prove by induction that $F_n > \phi^{n-2}$ for all $n \geq 3$. (One last time: Be careful to distinguish between what is *basis* and what are *cases* arising from the **induction step**!)