MA2441.03

Problem Set No. 4. (Relations, Functions; More on Induction)

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Date: June 14, 1999
Due: June 28, 1999

1. Prove that the composition of two 1-1 correspondences is a 1-1 correspondence.

Reminder. There are **three** issues to address.

2. (a) Prove that if a *total* relation R on a set A is *symmetric* and *transitive*, then it is also *reflexive*.

(b) By an appropriate example show that the assumption on totalness is *essential*.

3. Let S denote the set of strings over $\Sigma = \{1, 2, 3, +, \times, (,)\}$ defined as the *closure* of $\mathcal{I} = \{1, 2, 3\}$ under the operations $x, y \mapsto (x + y)$ and $x, y \mapsto (x \times y)$ for all strings x and y.

(a) Prove that every string x in S has equal numbers of "(" and ")" symbols in it.

(b) Prove the following claim for every $x \in S$: If $x = y * z$—where "$*$" denotes *concatenation*—and iff $\varepsilon \neq y \neq x$, then y contains *more* "("-symbols than ")"-symbols.

4. Let S be the set of strings over $\Sigma = \{0, 1\}$ obtained as the *closure* of $\mathcal{I} = \{01\}$† under a single operation on strings: $x \mapsto 0x1$ for all strings x.

Prove that $S = \{0^n 1^n : n \geq 1\}$, where v^n for a string v means $\underbrace{v * \cdots * v}_{n \text{ copies of } v}$ for any $n > 1$.

Reminder. There are **two** directions ($\subseteq$ and $\supseteq$).

† This is *not* a typo. $\mathcal{I}$ contains a single string: 01.