MA2441 3.0. Problem Set No. 5. (On Logic)
Dept. of Mathematics (Atkinson College)

Date: June 28, 1999
Due: July 12, 1999

Please note: The Final Take-Home Exam will be handed out in class, on Monday, July 12, 1999. **Don't miss it!**

It will be due (**in class—by 7:15pm**) on July 19, 1999 (**there will be absolutely no extensions**).

1. Prove by a *syntactic* argument that if $\vdash A \to B$, then $\vdash (\forall x)A \to (\forall x)B$ for any formulas A and B.

2. Using *resolution* prove the most general rule of "proof by cases", namely:

$$A \to B, C \to D \vdash A \vee C \to B \vee D$$

A proof-by-truth table will *not* be accepted in this exercise.

3. Prove by a *syntactic* argument that *if x is not free in B*, then

$$\vdash (\forall x)A \vee B \leftrightarrow (\forall x)(A \vee B)$$

for any wff A and B. (*Hint.* For $\to$ use proof by cases (problem 2) (you may also need to utilize problem 1 and $\forall$-introduction). For the other direction you will just need $\forall$-introduction.)

4. *True* or *False*? Give reasons.

(a) A propositional variable *must* have an *intrinsic* true or false meaning.

(b) Propositional variables can *only* denote mathematical formulas that contain no variables (e.g., $(\forall x)x = 4$ and $1 = 2 - 1$ are OK, but $x = 5$ and $x < y$ are not).

(c) Propositional Calculus studies the properties of "propositions".

5. Which of the following are "predicates"? Which are "atomic formulas"? *Explain. Use class-notes since the text is in error here!*

(a) $x \in y$
(b) $x = y$
(c) $\in$
(d) $=$
(e) P, where $P(x)$ stands for "$x > 3$ & $(\exists y)x = y^2$"
(f) $P(x)$ above.