MA2441.03

TAKE HOME EXAM

Dept. of Mathematics (Atkinson College)

Date: July 12, 1999
Due: July 19, 1999, **In class, by 7:15pm**

1. 10 MARKS Prove by *resolution* that
(i) $A \ \& \ (\neg B \to \neg A) \vdash B$, and
(ii) $\vdash (A \to B) \to ((B \to C) \to (A \to C))$

Every tool used in your proof must be explicitly acknowledged.

2. 10 MARKS For the arbitrary formula A of first order logic prove *syntactically with full annotation* that

$$\vdash (\exists x)(\forall y)A \to (\forall y)(\exists x)A$$

3. 5 MARKS Prove by induction on n that $5^n - 4n - 1$ is divisible by 16 for all $n \geq 1$.

4. 5 MARKS Prove that if $2^n - 1$ is a prime, then so is n.

5. 5 MARKS Let us define inductively a set of strings over the set $\{0, 1\}$ as the closure of $\mathcal{I} = \{\varepsilon\}$ (where ε denotes the EMPTY STRING) under the two *string* operations, $\mathcal{F}$:
(i) $x \mapsto x0$ and
(ii) $x \mapsto x1$.

Prove that $\mathrm{Cl}(\mathcal{I}, \mathcal{F}) = \{0, 1\}^*$.

6. 5 MARKS Let $f: A \to B$ and $g: B \to C$ be *total* functions such that $g \cdot f$ is 1–1 and f is *onto*. Show that g *must* be 1–1 under the circumstances.

Recall that $g \cdot f$ means $f \circ g$

7. 5 MARKS You are *given* that it is impossible to have $A \in B \in A$ for *any* sets A, B. Under the circumstances, *prove that*, for any sets and/or atoms x, y, a, b,

$$\{x, \{x, y\}\} = \{a, \{a, b\}\} \text{ implies } x = a \ \& \ y = b$$