MA3190.03

Problem Set No. 1

Dept. of Mathematics and Statistics

Date: January 21, 1999
Due: In two weeks

1. Suppose we defined "$((\exists x)\mathcal{A}[x])$ is true" to mean that for some value of x in the relevant domain, say c, $\mathcal{A}[c]$ is true. Under such definition, is $(\exists x)y < x$ true over $\mathbb{R}$? How about under the "normal" definition?

2. Let a be a set, and consider the class $b = \{x \in a : x \notin x\}$.
Show that, despite similarities with the Russell class R, b *is* a set.
Moreover, show that $b \notin a$.

3. Show that R (the Russell class)$= \mathbb{U}$.

4. Show that if a class $\mathbb{A}$ satisfies $\mathbb{A} \subseteq \mathbb{X}$ for all $\mathbb{X}$, then $\mathbb{A} = \emptyset$.

5. Without using foundation, show that $\emptyset \neq \{\emptyset\}$.

6. What is $\bigcap \emptyset$ (and why)?

7. For any set A, show that $\mathbb{U} - A$ is a proper class.

8. Show for any classes $\mathbb{A}, \mathbb{B}$, that $\mathbb{A} - \mathbb{B} = \mathbb{A} - \mathbb{A} \cap \mathbb{B}$.

9. For any classes $\mathbb{A}, \mathbb{B}$ show that $\mathbb{A} - (\mathbb{A} - \mathbb{B}) = \mathbb{B}$ iff $\mathbb{B} \subseteq \mathbb{A}$.

10. (1) Express $\mathbb{A} \cap \mathbb{B}$ using class difference as the only operation.
(2) Express $\mathbb{A} \cup \mathbb{B}$ using class difference/complement as the only operations.

11. Show that we cannot have $a \in b \in c \in \cdots \in a$.