MA3190.03

Problem Set No. 2

Dept. of Mathematics and Statistics

Date: Feb 11, 1999
Due: In three weeks

1. Show that for any class (not just set) $\mathbb{A}$, $\mathbb{A} \in \mathbb{A}$ is false.

2. (1) Show that $\mathbb{A} =$"the class of *all* sets that contain at least one element" can be defined by a class-term.
(2) Show that $\mathbb{A}$ is a proper class.

3. Attach the intuitive meaning to the statement that the set A has n (distinct) elements.

Show then by induction on n, that for $n \geq 0$, if A has n elements, then $\mathbf{P}(A)$ has 2^n elements.

4. Show (without the use of foundation) that $\big\{\{a\}, \{a, b\}\big\} = \big\{\{a'\}, \{a', b'\}\big\}$ implies $a = a'$ and $b = b'$.

5. For any sets x, y show that $x \cup \{x\} = y \cup \{y\} \to x = y$.
(*Hint*: Use foundation.)

6. For any $\mathbb{A}, \mathbb{B}$ show that $\emptyset = \mathbb{A} \times \mathbb{B}$ iff $\mathbb{A} = \emptyset$ or $\mathbb{B} = \emptyset$.

7. Show that $\mathbb{U}^3_M \subseteq \mathbb{U}^2_M$.

8. Let $\mathbb{F} : \mathbb{X} \to \mathbb{Y}$ be a function, and $\mathbb{A} \subseteq \mathbb{Y}$, $\mathbb{B} \subseteq \mathbb{Y}$. Prove
(a) $\mathbb{F}^{-1}[\mathbb{A} \cup \mathbb{B}] = \mathbb{F}^{-1}[\mathbb{A}] \cup \mathbb{F}^{-1}[\mathbb{B}]$
(b) $\mathbb{F}^{-1}[\mathbb{A} \cap \mathbb{B}] = \mathbb{F}^{-1}[\mathbb{A}] \cap \mathbb{F}^{-1}[\mathbb{B}]$
(c) if $\mathbb{A} \subseteq \mathbb{B}$, then $\mathbb{F}^{-1}[\mathbb{B} - \mathbb{A}] = \mathbb{F}^{-1}[\mathbb{B}] - \mathbb{F}^{-1}[\mathbb{A}]$.
Is this last equality true if $\mathbb{A} \not\subseteq \mathbb{B}$? Why?

9. Using only the axioms of union and separation, show that if a function $\mathbb{F}$ is a set, then so are both $\mathrm{dom}(\mathbb{F})$ and $\mathrm{ran}(\mathbb{F})$.

10. Show that if for a relation $\mathbb{S}$, both the range and the domain are sets, then $\mathbb{S}$ is a set.

11. Show for any relation S, that if S is a set then so is S^{-1}.