

MATH 6030.03

A/W 1990–2000

Date: Sep 21, 1999

Due: Soon

Problem Set No. 1

- 1) Why is “ $\neg\neg$ ” *not* a term? (Give a proof).
- 2) Why is $\mathcal{A}[x \leftarrow x]$ the same string as \mathcal{A} ? (Give a proof by induction on \mathcal{A}).
- 3) Prove that $\vdash t = s \rightarrow s = r \rightarrow t = r$, for any terms t , s , and r .
- 4) We have shown in class that if $\mathcal{F} + \neg\mathcal{A}$ is inconsistent, where \mathcal{A} is a sentence, then $\mathcal{F} \vdash \mathcal{A}$. Is this true for *any* wff \mathcal{A} (not necessarily closed)? Prove or disprove.
- 5) (*The proof-by-auxiliary-constant metatheorem*) Prove 4.23.
- 6) Prove the *Leibniz Rule*.
- 7) Prove that if $\mathcal{F} \vdash t_i = s_i$ for $i = 1, \dots, n$, and \bar{A} is the wff obtained from A by replacing some (possibly all) occurrences of t_i in A by s_i , then

$$\mathcal{F} \vdash A \leftrightarrow \bar{A}.$$

- 8) Prove that for any \mathcal{A} , $(\forall x)(\forall y)\mathcal{A} \vdash (\forall y)(\forall x)\mathcal{A}$.
Is it also the case that $\vdash (\forall x)(\forall y)\mathcal{A} \rightarrow (\forall y)(\forall x)\mathcal{A}$? Prove or disprove.