CSE 3402: Intro to Artificial Intelligence
Informed Search I

- Required Readings: Chapter 3, Sections 5 and 6, and Chapter 4, Section 1.

Heuristic Search.

- In uninformed search, we don't try to evaluate which of the nodes on the frontier are most promising. We never "look-ahead" to the goal.
  - E.g., in uniform cost search we always expand the cheapest path. We don't consider the cost of getting to the goal.
- Often we have some other knowledge about the merit of nodes, e.g., going the wrong direction in Romania.

Heuristic Search.

- Merit of a frontier node: different notions of merit.
  - If we are concerned about the cost of the solution, we might want a notion of merit of how costly it is to get to the goal from that search node.
  - If we are concerned about minimizing computation in search we might want a notion of ease in finding the goal from that search node.
  - We will focus on the "cost of solution" notion of merit.

Heuristic Search.

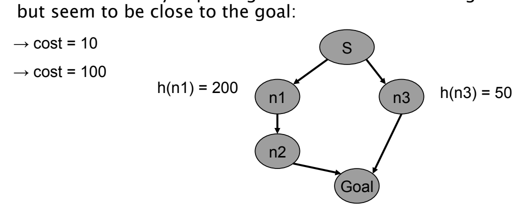
- The idea is to develop a domain specific heuristic function h(n).
- h(n) guesses the cost of getting to the goal from node n.
- There are different ways of guessing this cost in different domains. I.e., heuristics are domain specific.

Heuristic Search.

- Convention: If $h(n_1) < h(n_2)$ this means that we guess that it is cheaper to get to the goal from n_1 than from n_2.
- We require that
  - h(n) = 0 for every node n that satisfies the goal.
    - Zero cost of getting to a goal node from a goal node.

Using only h(n)
Greedy best-first search.

- We use h(n) to rank the nodes on open.
  - Always expand node with lowest h-value.
- We are greedily trying to achieve a low cost solution.
- However, this method ignores the cost of getting to n, so it can be lead astray exploring nodes that cost a lot to get to but seem to be close to the goal:



A* search

- Take into account the cost of getting to the node as well as our estimate of the cost of getting to the goal from n.
- Define
  - f(n) = g(n) + h(n)
    - g(n) is the cost of the path to node n
    - h(n) is the heuristic estimate of the cost of getting to a goal node from n.
- Now we always expand the node with lowest f-value on the frontier.
- The f-value is an estimate of the cost of getting to the goal via this node (path).

Conditions on h(n)

- We want to analyze the behavior of the resultant search.
- Completeness, time and space, optimality?
- To obtain such results we must put some further conditions on the heuristic function h(n) and the search space.

Conditions on h(n): Admissible

- $c(n1 \rightarrow n2) \geq \epsilon > 0$. The cost of any transition is greater than zero and can't be arbitrarily small.
- Let $h^*(n)$ be the cost of an optimal path from n to a goal node (∞ if there is no path). Then an admissible heuristic satisfies the condition
  - $h(n) \leq h^*(n)$
    - i.e. *h* always underestimates of the true cost.
- Hence
  - $h(g) = 0$
  - For any goal node "g"

Consistency/monotonicity.

- Is a stronger condition than $h(n) \leq h^*(n)$.
- A monotone/consistent heuristic satisfies the triangle inequality (for all nodes n1,n2):

$$h(n1) \leq c(n1 \rightarrow n2) + h(n2)$$

- Note that there might be more than one transition (action) between n1 and n2, the inequality must hold for all of them.
- Note that monotonicity implies admissibility. Why?

Intuition behind admissibility

- $h(n) \leq h^*(n)$ means that the search won't miss any promising paths.
  - If it really is cheap to get to a goal via n (i.e., both g(n) and h*(n) are low), then f(n) = g(n) + h(n) will also be low, and the search won't ignore n in favor of more expensive options.
  - This can be formalized to show that admissibility implies optimality.

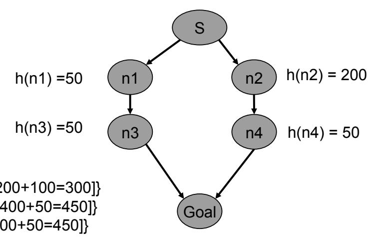
Intuition behind monotonicity

- $h(n1) \leq c(n1 \rightarrow n2) + h(n2)$
  - This says something similar, but in addition one won't be "locally" mislead. See next example.

Example: admissible but nonmonotonic

- The following *h* is not consistent since *h(n2)>c(n2→n4)+h(n4). But it is admissible.*

→ cost = 200
→ cost = 100



{S} → {n1 [200+50=250], n2 [200+100=300]}
→ {n2 [100+200=300], n3 [400+50=450]}
→ {n4 [200+50=250], n3 [400+50=450]}
→ {goal [300+0=300], n3 [400+50=450]}

We **do find** the optimal path as the heuristic is still admissible. **But** we are mislead into ignoring n2 until after we expand n1.

Consequences of monotonicity

1. The f-values of nodes along a path must be non-decreasing.

- Let <Start→ n1→ n2…→ nk> be a path. We claim that
  f(ni) ≤ f(ni+1)

- Proof:
  f(ni) = c(Start→ …→ ni) + h(ni)
  ≤ c(Start→ …→ ni) + c(ni→ ni+1) + h(ni+1)
  = c(Start→ …→ ni→ ni+1) + h(ni+1)
  = g(ni+1) + h(ni+1)
  = f(ni+1).

Consequences of monotonicity

2. If n2 is expanded after n1, then f(n1) ≤ f(n2)

Proof:

- If n2 was on the frontier when n1 was expanded,
  - f(n1) ≤ f(n2)

  otherwise we would have expanded n2.

- If n2 was added to the frontier after n1's expansion, then let n be an ancestor of n2 that was present when n1 was being expanded (this could be n1 itself). We have f(n1) ≤ f(n) since A* chose n1 while n was present in the frontier. Also, since n is along the path to n2, by property (1) we have f(n)≤f(n2). So, we have
  - f(n1) ≤ f(n2).

Consequences of monotonicity

3. When *n* is expanded every path with lower f-value has already been expanded.

- Assume by contradiction that there exists a path <Start, n0, n1, ni-1, ni, ni+1, …, nk> with f(nk) < f(n) and ni is its **last expanded** node.

- Then ni+1 must be on the frontier while n is expanded:
  a) by (1) f(ni+1) ≤ f(nk) since they lie along the same path.
  b) since f(nk) < f(n) so we have *f(ni+1) < f(n)*
  c) by (2) f(n) ≤ f(ni+1) since n is expanded before ni+1.
  * Contradiction from b&c!

Consequences of monotonicity

4. With a monotone heuristic, the first time A* expands a state, it has found the minimum cost path to that state.

- Proof:
  - * Let PATH1 = <Start, n0, n1, …, nk, n> be **the first** path to n found. We have f(path1) = c(PATH1) + h(n).
  - * Let PATH2 = <Start, m0,m1, …, mj, n> be another path to n found later. we have f(path2) = c(PATH2) + h(n).
  - * By property (3), f(path1) ≤ f(path2)
  - * hence: c(PATH1) ≤ c(PATH2)

Consequences of monotonicity

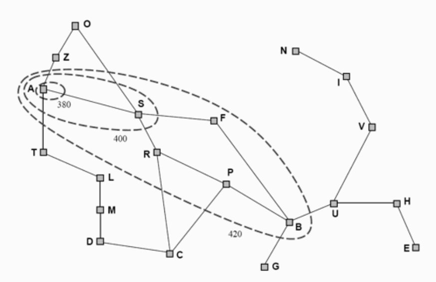
- Complete.
  - Yes, consider a least cost path to a goal node
    - SolutionPath = <Start→ n1→ …→ G> with cost c(SolutionPath)
    - Since each action has a cost ≥ ϵ > 0, there are only a finite number of nodes (paths) that have cost ≤ c(SolutionPath).
    - All of these paths must be explored before any path of cost > c(SolutionPath).
    - So eventually SolutionPath, or some equal cost path to a goal must be expanded.
- Time and Space complexity.
  - When h(n) = 0, for all n
    - h is monotone.
  - A* becomes uniform-cost search!
  - It can be shown that when h(n) > 0 for some n, the number of nodes expanded can be no larger than uniform-cost.
  - Hence the same bounds as uniform-cost apply. (These are worst case bounds).

Consequences of monotonicity

- Optimality
  - Yes, by (4) the first path to a goal node must be optimal.
- Cycle Checking
  - If we do cycle checking (e.g. using GraphSearch instead of TreeSearch) it is still optimal. Because by property (4) we need keep only the first path to a node, rejecting all subsequent paths.

Search generated by monotonicity

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Admissibility without monotonicity

- When "h" is admissible but not monotonic.
  - Time and Space complexity remain the same. Completeness holds.
  - Optimality still holds (without cycle checking), but need a different argument: don't know that paths are explored in order of cost.
- Proof of optimality (without cycle checking):
  - Assume the goal path <S,…,G> found by A* has cost bigger than the optimal cost: i.e. $C^* < f(G)$.
  - There must exists a node n in the optimal path that is still in the frontier.
  - We have: $f(n)=g(n)+h(n) \leq g(n)+h^*(n)=C^* < f(G)$
  - Therefore, f(n) must have been selected before G by A*. contradiction!

Admissibility without monotonicity

- No longer guaranteed we have found an optimal path to a node ***the first time*** we visit it.
- So, cycle checking might not preserve optimality.
  - To fix this: for previously visited nodes, must remember cost of previous path. If new path is cheaper must explore again.
- contours of monotonic heuristics don't hold.

Space problem with A* (like breath-first search):

IDA* is similar to Iterative Lengthening Search: It puts the newly expanded nodes in the front of frontier! Two new parameters:

- curBound (any node with a bigger f value is discarded)
- smallestNotExplored (the smallest f value for discarded nodes in a round) when frontier becomes empty, the search starts a new round with this bound.

Building Heuristics: Relaxed Problem

- One useful technique is to consider an easier problem, and let h(n) be the cost of reaching the goal in the easier problem.
- 8-Puzzle moves.
  - Can move a tile from square A to B if
    - A is adjacent (left, right, above, below) to B
    - and B is blank
- Can relax some of these conditions
  1. can move from A to B if A is adjacent to B (ignore whether or not position is blank)
  2. can move from A to B if B is blank (ignore adjacency)
  3. can move from A to B (ignore both conditions).

Building Heuristics: Relaxed Problem

- #3 leads to the misplaced tiles heuristic.
  - To solve the puzzle, we need to move each tile into its final position.
  - Number of moves = number of misplaced tiles.
  - Clearly h(n) = number of misplaced tiles $\leq$ the $h^*(n)$ the cost of an optimal sequence of moves from n.
- #1 leads to the manhattan distance heuristic.
  - To solve the puzzle we need to slide each tile into its final position.
  - We can move vertically or horizontally.
  - Number of moves = sum over all of the tiles of the number of vertical and horizontal slides we need to move that tile into place.
  - Again h(n) = sum of the manhattan distances $\leq h^*(n)$
    - in a real solution we need to move each tile at least that that far and we can only move one tile at a time.

Building Heuristics: Relaxed Problem

- The optimal cost to nodes in the relaxed problem is an admissible heuristic for the original problem!
  Proof: the optimal solution in the original problem is a (*not necessarily optimal*) solution for relaxed problem, therefore it must be at least as expensive as the optimal solution in the relaxed problem.
- Comparison of IDS and A* (average total nodes expanded):

Depth	IDS	A*(Misplaced)	A*(Manhattan)
10	47,127	93	39
14	3,473,941	539	113
24	---	39,135	1,641

Let h1=Misplaced, h2=Manhattan

- Does h2 **always** expand less nodes than h1?
  - Yes! Note that h2 dominates h1, i.e. for all n: h1(n)≤h2(n). From this you can prove h2 is faster than h1.
  - Therefore, among several admissible heuristic the one with highest value is the fastest.

Building Heuristics: Pattern databases.

- Admissible heuristics can also be derived from solution to **subproblems: Each state is mapped into a partial specification, e.g. in 15-puzzle only *position of specific tiles matters.***

- Here are goals for two sub-problems (called Corner and Fringe) of 15puzzle.

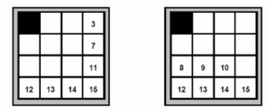


Fig. 2. The Fringe and Corner Target Patterns.

•By searching backwards from these goal states, we can compute the distance of any configuration of these tiles to their goal locations. We are ignoring the identity of the other tiles.

•For any state n, the number of moves required to get these tiles into place form a lower bound on the cost of getting to the goal from n.

Building Heuristics: Pattern databases.

- These configurations are stored in a database, along with the number of moves required to move the tiles into place.
- The maximum number of moves taken over all of the databases can be used as a heuristic.
- On the 15-puzzle
  - The fringe data base yields about a 345 fold decrease in the search tree size.
  - The corner data base yields about 437 fold decrease.
- Some times disjoint patterns can be found, then the number of moves can be added rather than taking the max.

Local Search

- So far, we keep the paths to the goal.
- For some problems (like 8-queens) we don't care about the path, we only care about the solution. Many real problem like Scheduling, IC design, and network optimizations are of this form.
- Local search algorithms operate using a single Current state and generally move to neighbors of that state.
- There is an objective function that tells the value of each state. The goal has the highest value (global maximum).
- Algorithms like Hill Climbing try to move to a neighbor with the highest value.
- Danger of being stuck in a local maximum. So some randomness can be added to "shake" out of local maxima.

Local Search

- Simulated Annealing: Instead of the best move, take a random move and if it improves the situation then always accept, otherwise accept with a probability <1. Progressively decrease the probability of accepting such moves.
- Local Beam Search is like a parallel version of Hill Climbing. Keeps K states and at each iteration chooses the K best neighbors (so information is shared between the parallel threads). Also stochastic version.
- Genetic Algorithms are similar to Stochastic Local Beam Search, but mainly use crossover operation to generate new nodes. This swaps feature values between 2 parent nodes to obtain children. This gives a hierarchical flavor to the search: chunks of solutions get combined. Choice of state representation becomes very important. Has had wide impact, but not clear if/when better than other approaches.