LE/EECS 4401/5326 3.0 Artificial Intelligence
GS/EECS 5326 3.0 Topics in Artificial Intelligence

Dept. of Electrical Eng. & Computer Sci.
York University
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Midterm Test — March 3, 2022

Duration: 80 minutes
No Aids Allowed
Total marks: 70

Name: SOLUTIONS

Student Number:

1)	/5
2)	/10
3)	/10
4)	/5
5)	/20
6)	/10
7)	/10
Total	/70

1. [5 points] For each of the following statements indicate whether it is *true* or *false*:

   (a) In general, for a first-order logic knowledge base KB and query ϕ, checking whether KB entails ϕ is *decidable*. F

   (b) In general, for a propositional logic knowledge base KB and query ϕ, checking whether KB entails ϕ is *decidable*. T

   (c) In general, checking whether a set of first-order Horn clauses is satisfiable is *decidable*. F

   (d) In general, for an $\mathcal{ALC}$ description logic knowledge base KB and query ϕ, checking whether KB entails ϕ is *decidable*. T

   (e) Resolution is *sound*, i.e., if a clause c can be derived by resolution from a set of clauses S, then S entails c. T

2. [10 points] Explain in a line or two each of the following:

a) closed-world asssumption

assuming that whenever an atom is not entailed by the KB, it must be false

b) Horn clause

a clause with at most 1 positive litteral

c) minimal entailment

entailment defined as truth in all most normal models of a theory

d) nonmonotonic reasoning

an inference relation where adding statements to the KB may invalidate previous entailments, ie., where $KB \models_i \alpha$ and $KB \cup \{B\} \not\models_i \alpha$ for some α + B

e) SLD resolution

a form of resolution used with Horn clauses

3. [10 points] Translate the following English sentences into first-order logic:

(a) Every person has a parent.

$\forall x. (\text{Person}(x) \supset \exists y.\ \text{Parent}(y, x))$

(b) No student failed all courses.

$\neg \exists x. (\text{Student}(x) \land \forall y. (\text{Course}(y) \supset \text{Failed}(x, y)))$

or

$\forall x. (\text{Student}(x) \supset \exists y. (\text{Course}(y) \land \neg \text{Failed}(x, y)))$

(c) There is a question that every student knows the answer to.

$\exists x. (\text{Question}(x) \land \forall y. (\text{Student}(y) \supset \text{KnowsAns}(x, y)))$

(d) All but one of the puppies are healthy.

$\exists x. (\text{Puppy}(x) \land \forall y. (\text{Puppy}(y) \land y \neq x \supset \text{Healthy}(y))$

(e) A professor is happy if he/she belongs to no committees.

$\forall x. (\text{Professor}(x) \land \neg \exists y. (\text{Committee}(y) \land \text{Belongto}(x, y)) \supset \text{Happy}(x))$

4. [5 points] Skolemizing a CNF formula does not preserve logical equivalence. For example, if we skolemize $\exists x P(x)$, we get $P(a)$, which is not logically equivalent to the original formula. Yet, skolemization works when one does resolution proofs. Briefly explain why.

- Skolemization preserves satisfiability
- When we do resolution refutation proofs we check (un-)satisfiability (and resolution refutation is complete)

5. [20 points] Suppose that we have the following knowledge base (KB) represented as a set of FOL sentences about three elephants, Sam, Clyde, and Oscar:

(1) $Pink(sam)$ Sam is pink.

(2) $Gray(clyde)$ Clyde is gray.

(3) $Likes(clyde, oscar)$ Clyde likes Oscar.

(4) $Pink(oscar) \vee Gray(oscar)$ Oscar is either pink or gray.

(6) $\neg Pink(oscar) \vee \neg Gray(oscar)$ Oscar is not both pink and gray.

(6) $Likes(oscar, sam)$ Oscar likes Sam.

(a) Prove that the KB does *not* entail that *Oscar is pink*. That is, show that there is an interpretation that satisfies the KB but does not satisfy this conclusion.

Let $\mathcal{A} = \langle D, I \rangle$ where

$D = \{ sam, clyde, oscar \}$

$I(x) = x$ for all $x \in D$

$I(Pink) = \{ sam \}$

$I(Gray) = \{ clyde, oscar \}$

$I(Likes) = \{ \langle clyde, oscar \rangle, \langle oscar, sam \rangle \}$

$\mathcal{A}$ trivially satisfies (1), (2), (3) and (6).

$\mathcal{A} \models (4)$ since $\mathcal{A} \models Gray(oscar)$.

$\mathcal{A} \models (5)$ since $\mathcal{A} \models \neg Pink(oscar)$.

But $\mathcal{A} \not\models Pink(oscar)$ since $I(Pink) \not\ni I(oscar)$.

(b) Using the definition of entailment in terms of interpretations, prove that the KB *entails* that *some grey elephant likes some pink elephant.* Give the representation of the query in first-order logic. Do not use resolution.

Take an arbitrary interpretation $\mathcal{I}$ and assume $\mathcal{I} \models KB$.

Case 1 if $\mathcal{I} \models Pink(oscar)$:

Then $\mathcal{I} \models Likes(oscar, sam)$

and $\mathcal{I} \models Grey(sam)$

So $\mathcal{I} \models \exists x (Grey(x) \land Pink(y) \land Likes(x,y))$

Case 2 if $\mathcal{I} \not\models Pink(oscar)$

Since $\mathcal{I} \models (4)$ then $\mathcal{I} \models Grey(oscar)$

$\mathcal{I} \models Pink(sam)$ and $\mathcal{I} \models Likes(oscar, sam)$

So $\mathcal{I} \models \exists x (Grey(x) \land Pink(y) \land Likes(x,y))$

Thus $\mathcal{I} \models \exists x (Grey(x) \land Pink(y) \land Likes(x,y))$

6. 7. [10 points] Use the tableau method for $\mathcal{ALC}$ described in Baader and Sattler's paper to check whether the following concept is satisfiable/consistent. Show the steps and rules that are used. If the concept is satisfiable give the model(s) (satisfying interpretation(s)) obtained by the method.

$$C_0 = (\underbrace{(\forall R.(\exists R.A))}_{C_1} \sqcap \underbrace{(\exists R.B)}_{C_2}) \sqcap \underbrace{(\forall R.((\forall R.\neg A) \sqcup (\forall R.\neg B)))}_{C_3}$$

$x_0 \cdot \{C_0\}$

$\rightarrow_{\sqcap} \rightarrow_{\sqcap}$

$x_0 \cdot \{C_0, C_1, C_2, C_3\}$

$\rightarrow_{\exists}$

$x_0 \cdot \{C_0, C_1, C_2, C_3\} \xrightarrow{R} x_1 \cdot \{B\}$

$\rightarrow_{\forall} \rightarrow_{\forall}$

$x_0 \cdot \{C_0, C_1, C_2, C_3\} \xrightarrow{R} x_1 \cdot \{B, \exists R.A, \underbrace{(\forall R.\neg A) \sqcup (\forall R.\neg B)}_{C_4}\}$

$\rightarrow_{\exists}$ $x_0 \cdot \{C_0, C_1, C_2, C_3\} \xrightarrow{R} x_1 \cdot \{B, \exists R.A, C_4\} \xrightarrow{R} x_2 \cdot \{A\}$

$\rightarrow_{\sqcup}$ $x_0 \cdot \{C_0, C_1, C_2, C_3\} \xrightarrow{R} x_1 \cdot \{B, \exists R.A, C_4, \forall R.\neg B\} \xrightarrow{R} x_2 \cdot \{A\}$

$\rightarrow_{\forall}$ $x_0 \cdot \{C_0, C_1, C_2, C_3\} \xrightarrow{R} x_1 \cdot \{B, \exists R.A, C_4, \forall R.\neg B\} \xrightarrow{R} x_2 \cdot \{A, \neg B\}$

no more rules are applicable

Satisfiable!

model is $\mathcal{I} = \langle D, \mathcal{I} \rangle$ where

$D = \{x_0, x_1, x_2\}$

$\mathcal{I}[A] = \{x_2\}$

$\mathcal{I}[B] = \{x_1\}$

$\mathcal{I}[R] = \{\langle x_0, x_1\rangle, \langle x_1, x_2\rangle\}$

7. [10 points]

a) What are the extension(s) of the default logic theory $\langle \mathcal{D}, \mathcal{F} \rangle$, where

$$\mathcal{D} = \{\langle\, TruckDriver(x) \Rightarrow BeerDrinker(x)\rangle\} \text{ and}$$

$$\mathcal{F} = \{Sailor(john), TruckDriver(bob)\}?$$

One extension

$\{\phi \mid \mathcal{F} \cup \{BeerDrinker(bob)\} \models \phi\}$

b) What are the extension(s) of the default logic theory $\langle \mathcal{D}, \mathcal{F} \rangle$, where

$$\mathcal{D} = \{\langle\, TruckDriver(x) \Rightarrow BeerDrinker(x)\rangle\} \text{ and}$$

$$\mathcal{F} = \{Sailor(john), (TruckDriver(bob) \vee TruckDriver(paul))\}?$$

one extension

$\{\phi \mid \mathcal{F} \models \phi\}$

c) What are the extension(s) of the default theory $\langle \mathcal{D}, \mathcal{F} \rangle$, where

$$\mathcal{D} = \{\langle \mathit{TruckDriver}(x) \Rightarrow \mathit{BeerDrinker}(x)\rangle, \langle \mathit{DoesYoga}(x) \Rightarrow \neg \mathit{BeerDrinker}(x)\rangle\}$$

and $\mathcal{F} = \{\mathit{TruckDriver}(john), \mathit{DoesYoga}(john), \mathit{TruckDriver}(bob)\}$?

2 extensions

$\{ \phi \mid \{ \mathcal{F} \cup \{ BeerDrinker(john), BeerDrinker(bob)\} \models \phi \}$

and

$\{ \phi \mid \{ \mathcal{F} \cup \{ \neg BeerDrinker(john), BeerDrinker(bob)\} \models \phi \}$