Assignment 1

Total marks: 75.

Out: February 4
Due: February 25 at 23:59

Note: Your report for this assignment should be the result of your own individual work. Take care to avoid plagiarism ("copying"). You may discuss the problems with other students, but do not take written notes during these discussions, and do not share your written solutions.

1. [30 points] Consider a situation described as follows:

   > Joe, Sally, Bill, and Ellen are the only members of the Elm Street Bridge Club. Joe is married to Sally. Bill is Ellen's brother. The spouse of every married person in the club is also in the club.

   From these facts, most people would be able to determine that Ellen is not married.

   a) Write some sentences in first-order logic that *represent* the facts given above. Use the notation in Brachman and Levesque's book for this question. Also provide a glossary where you indicate the intended meaning of your predicate, function, and constant symbols in English.

   b) *Prove* that the given facts do *not* entail that Ellen is not married, i.e., give an interpretation that satisfies the facts but falsifies the conclusion.

   c) Write some sentences in first-order logic that represent the additional knowledge that most people would have and *prove* that the augmented set of sentences now entails that Ellen is not married. *Your proof should use the definition of entailment and refer to interpretations; do not use resolution!*

2. [10 points] Determine whether the following first-order logic sentence is valid *using resolution*:

   $$\exists x.\forall y.\forall z.((P(y) \supset Q(z)) \supset (P(x) \supset Q(x)))$$

   Show the various steps of the conversion to clausal form and the resolution derivation. For each resolution step, state which two clauses are being resolved and the substitution used.

3. [15 points] Suppose that we have a TBox $\mathcal{T}$ containing the following $\mathcal{ALC}$ axioms:

$$C \sqsubseteq B$$
$$B \sqsubseteq A$$
$$A \sqsubseteq \exists r.D$$
$$A \sqsubseteq \forall r.E$$
$$D \sqsubseteq \neg C$$

   a) Say whether the TBox $\mathcal{T}$ is satisfiable. If it is, give a model (satisfying interpretation) of $\mathcal{T}$. If it is not, explain why there is no satisfying interpretation.

   b) Say whether the concept D is satisfiable with respect to $\mathcal{T}$. If it is, give an interpretation that shows this. If it is not, explain why there is no satisfying interpretation.

   c) Say whether the concept expression $C \sqcap D$ is satisfiable with respect to $\mathcal{T}$. If it is, give an interpretation that shows this. If it is not, explain why there is no satisfying interpretation.

4. [20 points] Use the tableau method for description logic $\mathcal{ALC}$ described in Baader and Sattler's paper to check whether the following concepts are satisfiable/consistent. Show the steps and rules that are used. If the concept is satisfiable give the model (satisfying interpretation) obtained by the method.

   a) $(\forall R.\forall R.\forall R.\forall R.\neg A) \sqcap (\forall R.\exists R.\forall R.\exists R.B)$
   $\sqcap (\exists R.\forall R.\exists R.\exists R.C) \sqcap (\forall R.\forall R.\forall R.\exists R.A)$

   b) $(\forall R.\exists R.((\forall R.A) \sqcup (\forall R.B) \sqcup (\forall R.C))) \sqcap (\forall R.\forall R.((\exists R.\neg A) \sqcup (\exists R.\neg B)))$
   $\sqcap (\forall R.\forall R.\exists R.\neg C) \sqcap (\exists R.C)$