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Efficient Duplication Free and Minimal Keyword Search in Graphs

Mehdi Kargar, *Student Member, IEEE,* Aijun An, *Member, IEEE,* and Xiaohui Yu, *Member, IEEE*

Abstract—Keyword search over a graph searches for a subgraph that contains a set of query keywords. A problem with most existing keyword search methods is that they may produce duplicate answers that contain the same set of content nodes (i.e., nodes containing a query keyword) although these nodes may be connected differently in different answers. Thus, users may be presented with many similar answers with trivial differences. In addition, some of the nodes in an answer may contain query keywords that are all covered by other nodes in the answer. Removing these nodes does not change the coverage of the answer but can make the answer more compact. The answers in which each content node contains at least one unique query keyword are called *minimal answers* in this paper. We define the problem of finding duplication-free and minimal answers, and propose algorithms for finding such answers efficiently. Extensive performance studies using two large real data sets confirm the efficiency and effectiveness of the proposed methods.

Index Terms—Keyword Search, Graph Data, Polynomial Delay, Approximation Algorithm.

1 Introduction

KEYWORD search is a well known method for extracting relevant knowledge from a set of documents in information retrieval. Given a graph where nodes are associated with text, keyword search over the graph finds a subgraph that contains a set of query keywords. Due to the fact that many types of data can be represented by graphs, keyword search over graphs has received much attention in recent years. Most of the work in this area find minimal connected trees (e.g, Steiner trees with the minimum sum of edge weights [1], [2], [3], [4], [5]) or subgraphs that minimize a proximity function (e.g., the sum of distances from the nodes in the answer to a center node [6]). However, these methods may generate many trees or subgraphs with the same set of content nodes (i.e., nodes containing at least one query keyword) even though these answers may have different intermediate nodes connecting the content nodes.

The following example illustrates the duplication problem for a tree-based method. Suppose the nodes in an input graph are web pages. Two nodes are connected by an edge if there is a link from one page to the other. Consider Figure 1. The user is interested in finding pages that contain keywords k_1 and k_2. Two nodes m_{k1} and n_{k1} contain keyword k_1 and another two nodes m_{k2} and n_{k2} contain keyword k_2. The left graph in the figure contains 4 trees that cover m_{k1} and m_{k2}, where each branch from m_{k1} to m_{k2} is a tree. The right graph contains a single tree that covers n_{k1} and n_{k2}. Assume that the weight on each edge is the same. According to the ranking function used in the tree approaches, the tree that contains n_{k1} and n_{k2} in the right graph is produced **after** the first four trees that cover m_{k1} and m_{k2} on the left, because it has more edges than the other four trees. However, all the four trees on the left have the same set of content nodes. Since the users usually want to see different groups of content nodes that are close to each other and might not be interested in browsing multiple relations to see how the nodes that contain input keywords are related to each other, the above search results might not be desirable[1]. Producing results with distinct sets of content nodes can prevent the search engine from overwhelming the user with many similar answers.

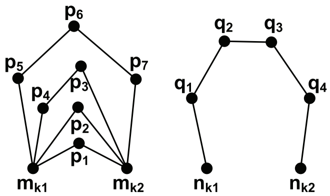

Fig. 1. Duplication problems with tree answers.

In addition to producing redundant results, current tree and graph-based methods may produce non-minimal answers. In other words, a content node in an answer may cover input keywords which are all covered by other content nodes. However, minimal answers may be preferred in some situations. Suppose that a customer wants to buy a set of items from stores and wants to find a set of stores that together have all the items he/she wants to buy. Assume that the information about the stores is stored in a graph,

• *The authors are with the Department of Computer Science and Engineering, York University, 4700 Keele Street, Toronto, Ontario M3J 1P3, Canada. E-mail: {kargar,aan}@cse.yorku.ca,xhyu@yorku.ca*

1. If a user wants to explore different relationships among the content nodes, the method in [7] can be used to produce a set of Steiner trees that connect a set of specified nodes together.

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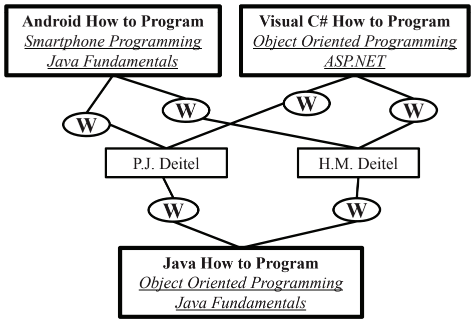
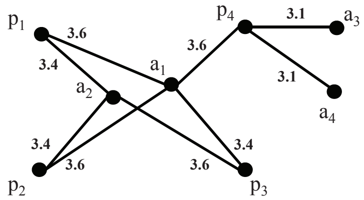


Fig. 2. A non-minimal answer for query: *Smartphone Programming*, *Java Fundamentals*, *Object Oriented Programming* and *ASP.NET* over a graph connecting books via authors.

where a node represents a store and contains the list of items that the store sells, and an edge between two nodes is weighted by the distance between the two stores. The customer issues a query specifying the set of items he/she wants to buy. It would be better that the search result is a list of stores in which each store has at least one unique item in the query that other stores do not have because there is no need to go to a store that does not have a unique item in the query. Another example is to determine required textbooks that together cover all the topics in a course. Assume that an online bookstore (e.g., *Amazon.com*) maintains its product information in an underlying graph where a node represents a book and contains the topics the book covers, and two books are connected by an edge if they share an author. Assume that the topics for a course are *Smartphone Programming*, *Java Fundamentals*, *Object Oriented Programming* and *ASP.NET*. A search over the graph allows us to find a set of books that not only covers all the topics but may also share the same author(s), which is preferred because the writing style of the books may be consistent. A possible answer to this query is shown in Fig. 2, where the three books share the same authors and together cover all the topics. But the topics covered by "Java How to Program" are also covered by the two other books. Thus, from the money-saving prospective it is not necessary to require the students to buy this book. In this type of applications, minimal answers are desired.

The following example shows the existing graph keyword search methods generate duplicate and non-minimal answers. Consider a small part of the DBLP dataset, which contains four authors and four papers. The paper titles, author names and a weighted graph that connect the authors and papers are shown in Figure 3. The edge weights are computed in the same way as in [6], [8]. Assume that the input keywords are k_1:*dynamic*, k_2:*fuzzy*, k_3:*logic*, k_4:*design* and k_5:*optimization*. Among all the subsets of the nodes, only $\{p_2, p_4\}$ covers all the input keywords and is also minimal. Other subsets either do not cover all the input keywords or are not minimal. The top-5 answers of the dynamic programming algorithm in [2] for



PID	Title	AID	Name
p_1	A Framework for Studying the Effects of **Dynamic** Crossover, Mutation, and Population Sizing in Genetic Algorithms	a_1	Michael A. Lee
p_2	**Dynamic** Control of Genetic Algorithms Using **Fuzzy Logic** Techniques	a_2	Hideyuki Takagi
p_3	Neural Networks and Genetic Algorithm Approaches to Auto **Design** of **Fuzzy** Systems	a_3	Henrik Esbensen
p_4	The **Design** of Hybrid Fuzzy Evolutionary Multiobjevtive **Optimization** Algorithms	a_4	Laurent Lemaitre

Fig. 3. A sample graph from the DBLP dataset.

TABLE 1
Steiner trees generated by dynamic programming.

No.	Root	Leaf Nodes (Content Nodes)
1	p_4	p_2, p_4
2	p_2	p_2, p_4
3	a_1	p_2, p_4
4	a_3	p_2, p_4
5	a_4	p_2, p_4

finding Steiner trees are given in Table 1, which shows that all the answers contain the same set of content nodes, although they have different roots connecting the content nodes. The top-5 answers of the BLINKS algorithm [4] are shown in Table 2, which shows that the sets of content nodes of the last three answers are exactly the same. In addition, none of the five answers is minimal. The top-5 answers of the community-finding method [6] are shown in Table 3. The second column of the table presents the association of each keyword with a node in the answer and the third column shows the set of content nodes. Clearly, some of these top-5 answers are duplicated and some of them are not minimal.

In this paper, we first propose a new approach to keyword search in graphs that produces duplication-free answers. Each answer produced by our approach has a unique set of content nodes. We also define *minimal answers*, in which each node contains at least one input keyword that other nodes do not. We propose two algorithms that convert an answer to a minimal answer. We prove that the problem of finding a minimal answer while minimizing the proximity function that we use is NP-hard. Thus, one of the algorithms we propose is a greedy algorithm that searches for a sub-optimal minimal answer. We prove that this greedy algorithm has a bounded approximation ratio. Finally, for finding top-k duplication-free and minimal answers, we propose two approaches. The first approach is faster but may miss some answers.

TABLE 2
Distinct root trees generated by BLINKS.

No.	Root	Leaf Nodes (Content Nodes)
1	p_2	p_2, p_3, p_4
2	p_4	p_1, p_2, p_4
3	a_1	p_1, p_2, p_3, p_4
4	p_3	p_1, p_2, p_3, p_4
5	a_2	p_1, p_2, p_3, p_4

TABLE 3
Answers from the community-finding method.

No.	Keyword-Node Association	Content Nodes
1	$(k_1, p_1), (k_2, p_4), (k_3, p_2), (k_4, p_4), (k_5, p_4)$	p_1, p_2, p_4
2	$(k_1, p_2), (k_2, p_2), (k_3, p_2), (k_4, p_4), (k_5, p_4)$	p_2, p_4
3	$(k_1, p_2), (k_2, p_4), (k_3, p_2), (k_4, p_4), (k_5, p_4)$	p_2, p_4
4	$(k_1, p_1), (k_2, p_2), (k_3, p_2), (k_4, p_4), (k_5, p_4)$	p_1, p_2, p_4
5	$(k_1, p_2), (k_2, p_2), (k_3, p_2), (k_4, p_3), (k_5, p_4)$	p_2, p_3, p_4

The second approach takes more time in theory but can produce all the answers if needed. Our extensive experiments show the efficiency and effectiveness of the proposed methods.

In the next section we discuss related work. In section 3, we give formal problem statements. In section 4, a procedure for finding duplication free answers in polynomial delay is presented. An algorithm for finding the best answer in each search space is given in section 5. Finding minimal answers is discussed in section 6. Experimental results are given in section 7. Section 8 concludes this work.

2 Related Work

Most of the approaches to keyword search over graphs find trees as answers. In [1], a backward search algorithm for producing Steiner trees is introduced. A dynamic programming procedure for finding Steiner trees in graphs are presented in [2]. In [3], the authors propose algorithms that produce Steiner trees with polynomial delay. The algorithms follow the Lawler's procedure [9]. Since finding Steiner trees is an NP-hard problem, producing trees with distinct roots is introduced in [5]. BLINKS improves the work in [5] by using an efficient indexing structure [4].

There are three methods that find subgraphs rather than trees for keyword search over graphs [10], [6], [8]. The first method finds r-radius Steiner graphs that contain all of the input keywords [10]. The second and third methods find multi-center communities or r-cliques as answers, respectively [6], [8]. The authors of [8] show that finding r-cliques are faster and more effective than finding communities. However, all of these approaches might produce duplicate and non-minimal answers.

Recently, the BROAD system is proposed to find diversified answers for keyword search on graphs [11]. The system is built on top of a keyword search engine and partitions the answer trees produced by the engine into dissimilar clusters. The dissimilarity between answers is measured based on the structural and semantic information of the given trees. A hierarchical browsing method is further proposed to help users navigate and browse the results. Our effort of finding duplication-free answers can be considered as a special case of finding diversified answers, where each answer must have a different set of content nodes. We find such "diversified" answers during the search process, while BROAD does it as a post-processing process. BROAD can be applied to the results of our method to further diversify the answers using the BROAD's dissimilarity measures.

Keyword search in graphs is closely related to finding a team of experts in social networks [12]. Given a set of required skills, the purpose is to find a set of experts that together cover all of the required skills and also be able to communicate efficiently. However, finding duplication free and minimal answers is not discussed in this area.

3 Problem Statement

Given a data graph whose nodes are associated with text and a query consisting of a set of keywords, the problem of keyword search in a graph is generally to find a subgraph that contains all or part of the keywords. The data graph can be directed or undirected. The edges and/or nodes may have weights on them. In this work, we consider undirected graphs with weighted edges, where two nodes are connected by an edge if there is a relationship between them and the edge weight represents the distance between the two nodes. It should be noted that our approach is adaptable to work with directed graphs[2].

Definition 1: **(Answer)** Given a graph G and a set of query keywords ($Q = \{k_1, k_2, \ldots, k_l\}$), an *Answer* to Q in G is a set of content nodes in G that together cover all of the input keywords in Q.

An *Answer* has a *weight* which can be defined according to the application need based on the weights of the edges in G that connect the nodes in the *Answer*. The above definition does not require the nodes in an *Answer* to be connected with each other either directly or indirectly in G, but $Answers$ with nodes connected to each other can be preferred over those with disconnected nodes by using a weight function.

Problem 1: **(Duplication free keyword search)** Given a graph G, an integer k and a set Q of query keywords, find top-k **unique** *Answers* of Q in G whose weights are optimal.

An *Answer* is *unique* if it appears at most once in the top-k list. The next definition deals with the minimality of the *Answer*.

2. Algorithms 1, 3, 5 and 6 proposed in this paper are independent of graph type. But the weight function and its related procedures (Algorithms 2 and 4) need to be adapted to work with directed graphs. For example, the weight of an answer can be defined using the weights of edges in both directions.

Definition 2: **(minAnswer)** Given a graph G and a set of query keywords ($Q = \{k_1, k_2, \ldots, k_l\}$), a *minAnswer* of Q in G is an *Answer* of Q in G in which each content node covers at least one query keyword that other content nodes do not cover.

Problem 2: **(Duplication free and minimal keyword search)** Given a graph G, an integer k and a set Q of input keywords, find top-k **unique minAnswers** of Q in G whose weights are optimal.

To focus on the generality of the above keyword search problems, we intentionally avoided defining the weight of an *Answer* in Definitions 1 and . Below we define a weight function, used in [8], to measure the *proximity* of the nodes in an *Answer*. Note that other weight functions can be used with our definitions. Also, most of the algorithms proposed in this paper (i.e., Algorithms 1, 3, 5 and 6) are independent of the weight function. Only Algorithms 2 and 4 depend on the weight function. [3]

Definition 3: **(sumDistance)** Suppose that the set of nodes in an $Answer$ in graph G is denoted as $V = \{v_1, v_2, \ldots, v_l\}$. The *sumDistance* of the $Answer$ is defined as

$$sumDistance = \sum_{i=1}^{l} \sum_{j=i+1}^{l} dist(v_i, v_j)$$

where $dist(v_i, v_j)$ is the shortest distance between v_i and v_j in G, i.e., the sum of weights on the shortest path between v_i and v_j in G [8].

When using $sumDistance$ to define the weight of an *Answer*, $Answers$ with smaller weights are considered to be better because the nodes in an *Answer* are closer to each other when its weight is smaller.

Note that our $Answer$ is a set of nodes. If the user wants to see the relationships among the nodes, algorithms in [8], [14] can be used to generate a Steiner tree connecting the nodes in an $Answer$. Generating a Steiner tree from an answer is much faster than generating a Steiner tree directly from the input graph [8]. Alternatively, a multi-center subgraph [6] can also be generated to reveal the relationships among the nodes in an $Answer$. Our system uses both methods for answer presentation.

3.1 An Overview of the Proposed Algorithms

In this paper, we propose six algorithms to solve Problems 1 and 2. Algorithm 1 is a general framework for generating top-k duplication-free answers by wisely dividing the search space. It calls Algorithm 2 (which is a 2-approximation algorithm for finding a single answer that minimizes $sumDistance$) to find top-k duplication-free answers in polynomial delay. Thus, Algorithms 1 and 2 together solve Problem 1.

3. Another weight function (i.e., $centerDistance$ used in [4], [6]) and their customized Algorithms 2 and 4 are given in [13].

To generate *minAnswers*, Algorithms 3 and 4 are proposed to convert the answers generated by Algorithm 2 into a *minAnswer*. Algorithm 3 does not optimize a weight function, while Algorithm 4 finds a *minAnswer* that also minimizes the $sumDistance$ function with a bounded approximation ratio. However, simply converting Algorithm 2's answer to a *minAnswer* with Algorithm 3 or 4 may lead to generation of duplicate answers in the top-k procedure.

To generate top-k *duplication-free minAnswers* (i.e., to solve Problem 2), Algorithms 5 and 6 are proposed to replace Algorithm 2 in Algorithm 1. Both algorithms are general frameworks for confining or dividing the search space to ensure minimality and no duplication in the top-k answers generated by Algorithm 1. They call a modified version of Algorithm 2 (which calls Algorithm 4) to generate a *minAnswer* that also minimizes $sumDistance$. The difference between Algorithms 5 and 6 is that Algorithm 5 is faster (completely polynomial) but may miss some answers, while Algorithm 6 is complete (i.e., it allows all the possible answers to be considered), but is a fixed-parameter tractable (FPT) algorithm. An overview of Algorithms 2-6 is given in Table 4.

4 Finding Top-k Duplication Free Answers in Polynomial Delay

Assume that the maximum number of nodes containing a query keyword in the input graph is m. Based on the definition of $Answer$, the total number of $Answers$ might be up to m^l, where l is the number of query keywords. Apparently, producing all of the $Answers$ may overwhelm the user since m and/or l can be large. Thus, it is important to produce top-k $Answers$ (or all the answers if fewer than k answers exist) in a ranked order. The efficiency of a search engine is commonly measured based on the delay between producing two consecutive answers. If this delay is polynomial based on the input data, the algorithm is called a polynomial delay algorithm [15], [3].

Our algorithm for producing top-k duplication-free answers is an adaption of Lawler's procedure [9] for finding top-k answers to discrete optimization problems. In Lawler's procedure, the search space is divided into disjoined sub-spaces. The best answer in each subspace is found and used to produce the current best global answer. The sub-space that produces the best global answer is further divided into sub-subspaces and the best answer among its sub-subspaces is used to compete with the best answers in other sub-spaces in the previous level to find the next best global answer. Two main issues in this procedure are how to divide a space into subspaces and how to find the best answer within a (sub)space. To have duplication free answers, the procedure for dividing the search space into sub-spaces must produce **disjoint** sub-spaces so that the same answer cannot be

TABLE 4
An overview of Algorithms 2-6.

Alg.	Complexity Type	Dup. Free	Minimal	Minimize $sumDistance$	Approx. Ratio	Complete	Time Complexity
Alg. 2	Polynomial	Yes	No	Yes, bounded approx.	2	Yes	$O(l^2 \times \|D_{max}\|^2)$
Alg. 3	Polynomial	No with Alg. 2	Yes	No	N/A	N/A	$O(n^2)$
Alg. 4	Polynomial	No with Alg. 2	Yes	Yes, bounded approx.	$(\log n)\frac{d_{max}}{d_{min}}$	N/A	$O(n^2)$
Alg. 5	Polynomial	Yes	Yes	N/A in general	N/A	No	$O(l^2 \times \|D_{max}\|^2)$
Alg. 6	FPT	Yes	Yes	N/A in general	N/A	Yes	$O((\prod_{i=1}^{s} \|K_i\|) \times l^2 \times \|D_{max}\|^2)$

Note: N/A means not applicable, l is the number of query keywords, $n(\leq l)$ is the number of nodes in the input answer for Alg. 4, d_{max} and d_{min} are the max and min distances between any pair of nodes in the input answer for Alg. 4, $s < l$, $\sum_{i=1}^{s} |K_i| < l$, $D_{max} \ll$ the number of nodes in graph.

TABLE 5
Dividing the search space into disjoint subspaces based on the best $Answer$ $\{a, b, c\}$.

Subspace	Inclusion set	Exclusion set
SB_1	$Inc_1 = \{a, b\}$	$Exc_1 = \{c\}$
SB_2	$Inc_2 = \{a\}$	$Exc_2 = \{b\}$
SB_3	$Inc_3 = \{\emptyset\}$	$Exc_3 = \{a\}$

generated from different sub-spaces.

Lawler's procedure has been used to generate top-k answers in graph keyword search in [6], [8], in which a search (sub)space is represented by $C_1 \times C_2 \times \cdots \times C_l$, where C_i is the set of nodes containing query keyword k_i, and the space is divided by taking away certain node(s) from C_i to form a subspace based on the best answer in the space being divided. A problem with this strategy is that a node taken away from C_i may appear in C_j (where $i \neq j$) if the node contains more than one query keyword (i.e., it belongs to more than one C_i for $1 \leq i \leq l$), and thus the same set of content nodes may be generated from different subspaces if a node contains more than one query keyword, although different answers have different keyword-node associations. Since we aim at generating unique sets of content nodes, a different strategy for dividing a search space is needed to avoid duplicate answers.

We first illustrate our idea using an example. Given a set of input keywords, we first use the $FindBestAnswer$ procedure (to be described later) to find the best answer $\{a, b, c\}$ in the input graph G, where a, b, and c are nodes in G. Then we divide the set of remaining answers to be found into three subsets: (1) the answers that contain a and b but no c, (2) the ones that contain a but no b, and (3) the ones that contain no a. Clearly, (1), (2) and (3) are disjoined, and they, together with $\{a, b, c\}$, comprise the set of all possible answers. Each subset has **constraints**, which can be represented using an **inclusion set** containing the nodes that must be included and an **exclusion set** containing the nodes that must be excluded[4]. Table 5 shows the constraints of these three subsets.

After dividing the search space into disjoint subsets based on the global best answer, the best $Answer$ in each subspace is found using the $FindBestAnswer$ procedure. These best $Answers$ are inserted into a priority queue, where the $Answers$ are ranked in ascending order of their weights. Obviously, the second best $Answer$ is the one at the top of the priority queue. Suppose that this $Answer$ is taken from SB_2 and contains p content nodes. After returning the second best answer, SB_2 is divided into p subspaces in the way similar to the one shown in Table 5. In each subspace, the best $Answer$ is found and is added to the priority queue. At this state, the priority queue has $2 + p$ elements: two elements from the first step and p elements from this new step[5]. Then, the top $Answer$ is returned and removed from the queue, its corresponding space is divided into subspaces and the best $Answer$ (if any) in each new subspace is added to the queue. This procedure continues until the priority queue becomes empty or top-k $Answers$ are found.

The pseudocode of this procedure for enumerating top-k $Answers$ is described in Algorithm 1. The algorithm first computes the set C of nodes that contain at least one input keyword. This can be easily done using a pre-built inverted index. In line 5, procedure $FindBestAnswer$ (to be described in the next section) is called to find the best answer from the whole search space (i.e., C). It takes input graph G, query Q, C, an inclusion set and an exclusion set as input, and returns the best answer in the search space specified by C. Since the first best answer is found in the whole search space, empty inclusion and exclusion sets are passed to the procedure in line 5. If the best answer exists (i.e., $A \neq$ NULL), A, together with the inclusion and exclusion sets (the constraints for the space from which A is generated), are inserted into $Queue$ in line 7. The $Queue$ is maintained in the way that its elements are ordered in ascending order of their weights. The while loop starting at line 8 is executed until the $Queue$ becomes empty or k answers have been outputted. In line 9, the top of the $Queue$ is removed, which contains the best answer (A) in the $Queue$ and its inclusion (Inc) and exclusion (Exc) sets. The answer in A is outputted. Then, if the number of answers has not reached k, the nodes in A are assigned to $n_1, n_2 \ldots n_p$ where p is the number of nodes in A. In lines 15-21, p new inclusion and

4. The idea of using the inclusion and exclusion sets to represent constraints is inspired by [16]. However, the constraints in [16] are described using *edges* (instead of *nodes* as in our approach) for finding a different type of answers.

5. This assumes that all of the subspaces contain at least one $Answer$. In some cases, the subspace does not have any $Answer$.

Algorithm 1 Generate Duplication Free Top-k $Answers$

Input: the input graph G; the query $Q = \{k_1, k_2, \ldots, k_l\}$; k
Output: the set of top-k ordered $Answers$ printed with polynomial delay

$C \leftarrow$ an empty set for storing content nodes
for $i \leftarrow 1$ **to** l **do**
    add the nodes in G containing k_i to C
$Queue \leftarrow$ an empty priority queue
$A \leftarrow$ **FindBestAnswer**(G, Q, C, $\emptyset$, $\emptyset$)
if $A \neq$ NULL **then**
    insert $\langle A, \emptyset, \emptyset\rangle$ into $Queue$
while $Queue \neq \emptyset$ **do**
    $\langle A, Inc, Exc\rangle \leftarrow$ top element of $Queue$
    print(A)
    $k \leftarrow k - 1$
    if $k = 0$ **then**
        return
    $\{n_1, n_2, \ldots, n_p\} \leftarrow$ content nodes of A
    for $i \leftarrow 1$ **to** p **do**
        $Inc_i \leftarrow Inc \cup \{n_1, \ldots, n_{p-i}\}$
        $Exc_i \leftarrow Exc \cup \{n_{p-i+1}\}$
        if $Inc_i \cap Exc_i = \emptyset$ **then**
            $A_i \leftarrow$ **FindBestAnswer**(G, Q, C, Inc_i, Exc_i)
            if $A_i \neq$ NULL **then**
                insert $\langle A_i, Inc_i, Exc_i\rangle$ into the right place of $Queue$ according to A_i's weight

exclusion sets are produced based on the nodes in A and the inclusion and exclusion sets for the space A was generated from. The new subspaces are specified by these new constraints. For each new subspace, if the intersection of its inclusion and exclusion sets is empty, the best answer is found and it is inserted into the $Queue$ with the constraints of its related subspace. Clearly, if procedure $FindBestAnswer$ runs in polynomial time, Algorithm 1 produces answers with polynomial delay.

Since for each best answer A the union of the sub-spaces created based on A plus answer A itself is the same as the search space from which A is found, no answer is excluded from search spaces in the next iterations. Thus, Algorithm 1 produces top-k or all answers (if fewer than k answers exist) if $FindBestAnswer$ finds the best answer in a search (sub)-space. In addition, the sub-spaces produced based on answer A are all disjoint and none of them contains A. Therefore, they do not lead to the same answer and the set of produced answers is duplication free. In addition, this duplication free search procedure is independent of the procedure for finding the best answer and the weight function used to measure the quality of an answer.

5 Finding the Best Answer in Each Search Space

Algorithm 1 calls the $FindBestAnswer$ procedure to find the best answer in a search space specified by a set of content nodes and the constraints (i.e., the inclusion and exclusion sets). The best answer must contain the nodes in the inclusion set, exclude the nodes in the exclusion set and also have an optimal weight. Depending on the weight function used, $FindBestAnswer$ can be designed differently. Below, we present an algorithm that produces an answer satisfying the constraints and minimizing the $sumDistance$ function.

Algorithm 2 FindBestAnswer minimizing the $sumDistance$ function

Input: the input graph G; the query Q; the set of content nodes C; the set of inclusion nodes Inc; the set of exclusion nodes Exc
Output: the best (approximate) $Answer$ satisfying both Inc and Exc constraints

$Cov \leftarrow$ set of keywords covered by Inc
$\{k_1, k_2, \ldots, k_t\} \leftarrow \{Q - Cov\}$
for $i \leftarrow 1$ **to** t **do**
    $D_i \leftarrow$ nodes of C having keyword k_i and $\notin Exc$
$D \leftarrow \bigcup_{i=1}^{t} D_i$
$F \leftarrow Inc \cup D$
if $F = \emptyset$ **then**
    return NULL
$leastWeight \leftarrow \infty$
$bestAnswer \leftarrow$ NULL
for each node f_i in F **do**
    $weight \leftarrow 0$
    $answer \leftarrow \emptyset$
    for each node n_j in Inc **do**
        $weight \leftarrow weight + d(f_i, n_j)$
        $answer = answer \cup \{n_j\}$
    for $j \leftarrow 1$ **to** t **do**
        $dist \leftarrow \infty$
        $nearest \leftarrow$ NULL
        for each node d_k in D_j **do**
            if $d(f_i, d_k) < dist$ **then**
                $dist = d(f_i, d_k)$
                $nearest = d_k$
        if $nearest \notin answer$ **then**
            $weight \leftarrow weight + dist$
            $answer = answer \cup \{nearest\}$
    if $weight < leastWeight$ **then**
        $leastWeight \leftarrow weight$
        $bestAnswer \leftarrow answer$
return $bestAnswer$

In [8] we proved that minimizing $sumDistance$ is an NP-hard problem, and proposed an approximation algorithm that finds an answer with an approximation ratio of 2. The search space in that algorithm is a Cartesian product $C_1 \times C_2 \times \cdots \times C_l$, where C_i is a subset of nodes containing keyword k_i and excluding certain nodes. However, a node excluded from C_i may appear in C_j if the node contains both k_i and k_j. Since our answers must completely exclude the nodes specified by the exclusion set, we modify the algorithm in [8] to consider the constraints specified by the inclusion and exclusion sets.

The pseudo-code of the modified algorithm, $FindBest\text{-}Answer$, is presented in Algorithm 2. It takes an input graph G, a query Q, a set of content nodes C, and the inclusion and exclusion sets (Inc and Exc) as input and produces the best (approximate) $Answer$ as output in polynomial time. The algorithm approximates the $sumDistance$ of an answer using the sum of distances from each node in the answer to a center node within the answer. In the pseudo-code, set F is the search space, which consists of all the nodes in the inclusion set and the set of content nodes containing the query keywords not covered by the inclusion set and not belonging to the exclusion set. In the code, D_i is the set of nodes that contain keyword k_i (which is not covered by the inclusion set) but do not belong to the exclusion set. For each node f_i in F, an answer is formed by using f_i as the center

Algorithm 3 ConvertToMinAnswerGeneral - General Procedure

Input: the set of content nodes as $Answer$; the query Q
Output: a $minAnswer$

```
for each node n_i in Answer do
    K = ∅
    for j ← 1 to i − 1 do
        K = K ∪ keywords(n_j)
    for j ← i + 1 to size(Answer) do
        K = K ∪ keywords(n_j)
    if keywords(n_i) ⊆ K then
        remove n_i from Answer
return Answer
```

and including all the nodes in the inclusion set and adding the node in each D_i that is closest to f_i. The final answer is the one with the least sum of distances between each node in the answer and its center. In the code, $d(x, y)$ is the shortest distance between nodes x and y, which can be efficiently obtained by consulting a pre-built index (described in [8])[6].

Clearly, the answer produced by this algorithm satisfies the inclusion and exclusion constraints. Since all the nodes in F have been considered as a center candidate, it can be proved that the $sumDistance$ of the produced answer is no more than $\frac{2\times(l-1)}{l}\times$ the $sumDistance$ of an optimal answer, where l is the number of query keywords. Thus, the produced answer has a weight that is at most twice that of an optimal answer. The proof is similar to the one in [8]. We omit it here due to the space limit. The complexity of this algorithm is $O(|F| \times l \times |D_{max}|)$ where $|F|$ is the size of the set F, l is the number of query keywords and $|D_{max}|$ is the maximum size of D_i for $1 \leq i \leq t$. Since $|F| \leq (l \times |D_{max}|) + |Inc|$ and $|Inc| \leq l-1$, $|F| = O(l\times|D_{max}|)$. Thus, the complexity of Algorithm 2 is $O(l^2 \times |D_{max}|^2)$.

6 Finding Minimal Answers

Some of the $Answers$ returned by Algorithm 2 and existing algorithms may not be a $minAnswer$. That is, the input keywords in some nodes of an $Answer$ may all be covered by other nodes in the answer. If these nodes are removed from the answer, the remaining set of nodes still covers all the input keywords. Below we first present two algorithms for converting an $Answer$ to a $minAnswer$. However, the converted $minAnswer$ may violate the inclusion constraint for finding duplication-free answers. We then propose two approaches to solve the problem.

6.1 Generating Minimal Answers

The problem of finding a minimal answer from an *Answer* can be solved in polynomial time as shown in Algorithm 3. The algorithm checks each node in the *Answer* to see if the input keywords the node contains are all covered by other nodes. If yes, it removes the node. The complexity of this algorithm is $O(n^2)$ where n is the number of nodes in the input $Answer$.

Lemma 1: Algorithm 3 produces a $minAnswer$ in which each node contains at least one unique input keyword. In addition, all the input keywords are covered in the $minAnswer$.

Proof: The proof is omitted due to the space limit, but is given in [13]. □

An $Answer$ may contain multiple $minAnswers$. The answer returned by Algorithm 3 may not be optimal with respect to a weight function such as $sumDistance$. Below we first prove that the problem of finding a $minAnswer$ with the minimum $sumDistance$ is an NP-hard problem, and then present a greedy algorithm to solve the problem.

Theorem 1: The problem of producing a $minAnswer$ from an $Answer$ while minimizing $sumDistance$ is NP-hard.

Proof: We prove the theorem by a reduction from the *set cover* problem. Given a set of m elements (universe) and n sets whose union is the universe, the *set cover* problem is to identify the *smallest* number of sets whose union still contains all elements in the universe. Consider the set of input keywords in our problem as a universe. The nodes in an $Answer$ can be considered as the sets of keywords whose union is the universe because they cover all the input keywords. Assume that the shortest distance between each pair of nodes in an $Answer$ is the same. Then finding a $minAnswer$ from the $Answer$ is equivalent to finding the minimal number of nodes that cover all the input keywords (i.e., the universe). This is because a $minAnswer$ with a smaller number of nodes has a smaller $sumDistance$ when the shortest distance between each pair of nodes is the same. Since the set cover problem is NP-hard [17], finding a $minAnswer$ while minimizing $sumDistance$ is NP-hard. □

Since the problem is NP-hard, we design a greedy algorithm to find a $minAnswer$ that may be sub-optimal in minimizing $sumDistance$. The algorithm is presented in Algorithm 4. It first uses a greedy set-covering procedure (Lines 1-6) to reduce the number of nodes in $Answer$ while still covering all the input keywords. The procedure chooses nodes to form an answer A as follows: at each stage, choose the node that contains the largest number of uncovered keywords. However, A may not be a $minAnswer$ because the above procedure is a greedy procedure for minimizing the number of nodes. Thus, we further sort the nodes in A based on their sum of distances to other nodes in descending order, and then call $ConvertToMinAnswerGeneral$ (i.e., Algorithm 3) to convert A into a $minAnswer$.

The complexity of the algorithm is $O(n^2)$, where n is the number of nodes in the input $Answer$. Also, since the set-covering procedure (Lines 1-6) chooses nodes from $Answer$ until all the input keywords are covered and Lemma 1 states that the $minAnswer$

6. Using a pre-built index to obtain the shortest distance between nodes has been used in [10], [6], [8].

Algorithm 4 ConvertToMinAnswer - Greedy Procedure for Minimizing $sumDistance$

Input: the set of content nodes as $Answer$; the query Q
Output: a $minAnswer$ with (sub)optimal $sumDistance$

1: $A \leftarrow \emptyset$
2: **while** $Q \neq \emptyset$ **do**
3:     **select** a node $n \in Answer$ that maximizes $|\mathbf{keywords}(n) \cap Q|$
4:     $Answer \leftarrow Answer - \{n\}$
5:     $Q \leftarrow Q - \mathbf{keywords}(n)$
6:     $A \leftarrow A \cup \{n\}$
7: **for** each node n_i in A **do**
8:     calculate n_i's sum of distances to all the other nodes in A
9: **sort** nodes in A based on their sum of distances to other nodes in descending order and put them in a list T.
10: $minAnswer = ConvertToMinAnswerGeneral(T, Q)$
11: **return** $minAnswer$

produced by $ConvertToMinAnswerGeneral$ covers all the input keywords, the $minAnswer$ produced by this algorithm covers all the input keywords.

Theorem 2: Algorithm 4 generates a $minAnswer$ that minimizes $sumDistance$ with the approximation ratio of $(\log n)\frac{d_{max}}{d_{min}}$ where n is the number of nodes in the input $Answer$ and d_{max} and d_{min} are the maximum and minimum distances between any pair of nodes in $Answer$.

Proof: Assume that the number of nodes of an optimal $minAnswer$ that minimizes $sumDistance$ is opt_n and the number of nodes of the $minAnswer$ produced by Algorithm 4 is $approx_n$. Also assume that the number of nodes of an optimal answer that minimizes the number of nodes (which is the objective of the set cover problem) is opt_{scn} and the number of nodes of the approximate answer produced by lines 1-6 (i.e., the greedy set cover procedure) is $approx_{scn}$. It has been proved that the number of nodes of the answer obtained by the greedy set cover algorithm is at most $\log n$ times that of the optimal answer [17], where n is the number of nodes in the input $Answer$. That is, $approx_{scn} \leq \log n \times opt_{scn}$. Since the later steps of Algorithm 4 may further reduce the number of nodes from the answer generated by the greedy set-cover procedure, $approx_n \leq approx_{scn}$. Also, it is obvious that $opt_{scn} \leq opt_n$. Thus, we have $approx_n \leq \log n \times opt_n$. For a query with l keywords, the $sumDistance$s of the optimal and the approximation answers satisfy the following inequalities: 1) $sumDistance_{opt} \geq [\binom{l}{2} - l + opn_n] \times d_{min}$ and 2) $sumDistance_{approx} \leq [\binom{l}{2} - l + approx_n] \times d_{max}$ where d_{max} and d_{min} are the maximum and minimum distances between any pair of nodes in the $Answer$, respectively. Since $approx_n \leq \log n \times opt_n$, we have:

$$\frac{sumDistance_{approx}}{sumDistance_{opt}} \leq \frac{[\binom{l}{2} - l + (\log n \times opt_n)] \times d_{max}}{[\binom{l}{2} - l + opn_n] \times d_{min}}$$

Therefore, the following can be easily derived:

$$\frac{sumDistance_{approx}}{sumDistance_{opt}} \leq (\log n)\frac{d_{max}}{d_{min}}. \qquad \square$$

It should be noted that Algorithm 4 is guaranteed to generate a $minAnswer$. The approximation is in terms of minimizing the weight of $minAnswers$.

Since the weight of a $minAnswer$ may be smaller than that of the $Answer$ the $minAnswer$ is generated from, Algorithm 4 should be called after line 26 of Algorithm 2 using $answer \leftarrow ConvertToMinAnswer(answer, Q)$. After that, the weight of the answer should be updated as well. Thus, in Algorithm 2 the generated $minAnswer$ of each candidate is used to compete with the $minAnswers$ of other candidates so that the $minAnswer$ with the smallest weight among the candidates can be returned by Algorithm 2.

Since the number of nodes in $Answer$ is at most the number of input keywords, the time complexity of Algorithm 2 becomes $O(|F| \times (|D_{max}| \times l + l^2))$, where l is the number of input keywords, $|D_{max}|$ is the maximum size of D_i (the set of the nodes containing keyword k_i) and $|F|$ is the size of set F. As we discussed in previous section, $|F| = O(l \times |D_{max}|)$. Therefore, the time complexity of Algorithm 2 becomes $O(l^2 \times D_{max} \times (D_{max} + l))$. Since l can be much smaller than $|D_{max}|$ ($l \ll |D_{max}|$), time complexity of Algorithm 4 is the same as Algorithm 2 and is equal to $O(l^2 \times |D_{max}|^2)$.

6.2 Producing Top-k / All Minimal Answers

To generate all or top-k duplication-free $minAnswers$, Algorithm 1 is needed to divide the search space and call Algorithms 2 and 4 to find a $minAnswer$ in each subspace. This procedure works fine for finding the first best $minAnswer$ in the whole search space. However, for finding subsequent answers, the search space is divided into subspaces, each with inclusion and exclusion constraints, and the best answer from each subspace is generated to compete for the next best answer. This requires that the $minAnswer$ generated from each subspace contains all the nodes in the inclusion set of that subspace. However, when generating a $minAnswer$ from an $Answer$ and when the inclusion set is not empty, Algorithm 4 may delete some of the inclusion nodes if their keywords are covered by other nodes in the $Answer$. This may lead to generating duplicate answers by Algorithm 1. An example that illustrates the problem is given in [13].

To solve this problem, we change Algorithm 2 so that all the inclusion nodes in the $Answer$ produced by the algorithm must contain at least one unique input keyword. In this way, the inclusion nodes in the answer cannot be removed when converting the $Answer$ to a $minAnswer$. Below we propose two approaches that use this strategy. The first one is called the *incomplete* approach. It is faster but may miss some answers. The second approach is called the *complete* approach. It considers all the answers but has higher time complexity than the first approach. The algorithms for both approaches are named $FindMinimalAnswer$ below. They are called in Algorithm 1 at the places where Algorithm 2 was called. Both approaches are independent of the weight function used to measure the quality of the answer.

6.2.1 *Incomplete Approach*

Based on the way the search space is divided in Alg. 1, the nodes in the inclusion set of a subspace are part of a previously-generated $minAnswer$. Thus, each node in an inclusion set has at least one unique keyword among other nodes in the set. If in Alg. 2 each D_i contains only the nodes that do not contain any keyword that an inclusion node contains, the inclusion nodes will keep their uniqueness and will not be removed when converting the $Answer$ to a $minAnswer$. This is the idea of the incomplete approach.

The pseudo-code of this approach is presented in Algorithm 5. Its inputs are the same as the ones for Algorithm 2. It first collects the keywords covered by the inclusion nodes into $CovKeywords$. Then it calls procedure $FindBestAnswerCovConstraint$ to generate a $minAnswer$. Procedure $FindBestAnswerCovConstraint$ is similar to procedure $FindBestAnswer$ (i.e., Algorithm 2) with two differences. The first difference is that in addition to other inputs, it also takes set $CovKeywords$ as input and in line 4 of procedure $FindBestAnswer$ the algorithm also excludes from D_i all the nodes that contain a keyword in $CovKeywords$. Since D_is store the candidate nodes to be added to the $answer$, this exclusion guarantees that no node with a keyword in $CovKeywords$ is added to the $answer$. The second difference is that the procedure calls Algorithm 4 after line 26 to convert a candidate answer to a $minAnswer$ and then calculates the weight of the $minAnswer$. The best $minAnswer$ is returned. In section 6.1, we have showed that calling Algorithm 4 within Algorithm 2 does not change the complexity of Algorithm 2. Thus, the time taken by Algorithm 5 is the same as Algorithm 2 and is equal to $O(l^2 \times |D_{max}|^2)$.

Algorithm 5 FindMinimalAnswer, Incomplete Approach

Input: the input graph G; the query Q; the set of content nodes C; the set of inclusion nodes Inc; the set of exclusion nodes Exc
Output: the best *minAnswer* satisfying both Inc and Exc constraints

$CovKeywords \leftarrow$ set of keywords covered by Inc
$minAnswer \leftarrow$ **FindBestAnswerCovConstraint**(G, Q, C, Inc, Exc, $CovKeywords$)
return $minAnswer$

However, Algorithm 5 may miss some answers because it puts a too strong constraint on the search space and removes some good candidate nodes.

6.2.2 *Complete Approach*

To solve the missing-answer problem of the *incomplete* approach, we propose the *complete* approach. Since each node in the inclusion set has at least one unique keyword, we first compute the set of unique keywords for each node in the inclusion set and then calculate the Cartesian product of these sets. For example, if $Inc = \{a, b\}$ and a and b uniquely contain $\{k_1, k_2\}$ and $\{k_3, k_4\}$ respectively, the Cartesian product of $\{k_1, k_2\}$ and $\{k_3, k_4\}$ is $\{k_1, k_3\}$, $\{k_1, k_4\}$, $\{k_2, k_3\}$ and $\{k_2, k_4\}$. Then, for each set s in the Cartesian product, procedure $FindBestAnswerCovConstraint$ is called with s as the input value for $CovKeywords$ to generate a $minAnswer$ whose non-inclusion nodes do not contain any keyword in s. Among all of the $minAnswers$ (each generated based on an element in the Cartesian product), the best $minAnswer$ is returned as the solution.

The pseudo-code of the complete approach is presented in Algorithm 6. It first gets the set of inclusion nodes as $\{n_1, n_2, \ldots, n_s\}$. Then, for each content node $n_i \in Inc$, it gets the unique keywords covered by n_i and stores them in K_i. The Cartesian product of $\{K_1, K_2, \ldots, K_s\}$ is calculated and stores in $CKeywordSet$ in line 3. For each member $CovKeywords_i$ of $CKeywordSet$, a $minAnswer$ is found by calling $FindBestAnswerCovConstraint$ in line 8. Procedure $FindBestAnswerCovConstraint$ is the same as the one used in the *incomplete* approach. It finds a $minAnswer$ and makes sure that its non-inclusion nodes do not contain any keywords in $CovKeywords_i$. If the $minAnswer$ is not NULL and its weight outperforms previous minimal answers, $leastWeight$ and $bestMinAnswer$ are updated accordingly. The algorithm returns the $minAnswer$ with the smallest weight among all the $minAnswers$ corresponding to the members of the Cartesian product.

Algorithm 6 FindMinimalAnswer, Complete Approach

Input: the input graph G; the query Q; the set of content nodes C; the set of inclusion nodes Inc; the set of exclusion nodes Exc
Output: the best *minAnswer* satisfying both Inc and Exc constraints

1: $\{n_1, n_2, \ldots, n_s\} \leftarrow$ set of nodes of Inc
2: $\forall i, 1 \leq i \leq s, K_i \leftarrow$ unique keywords of n_i
3: $CKeywordSet \leftarrow$ Cartesian product of $\{K_1, K_2, \ldots, K_s\}$
4: $leastWeight \leftarrow \infty$
5: $bestMinAnswer \leftarrow$ NULL
6: **for** $i \leftarrow 1$ **to size**($CKeywordSet$) **do**
7: $\quad CovKeywords_i \leftarrow CKeywordSet$.**get**($i$)
8: $\quad minAnswer \leftarrow$ **FindBestAnswerCovConstraint**(G, Q, C, Inc, Exc, $CovKeywords_i$)
9: $\quad$ **if** $minAnswer \neq$ NULL **then**
10: $\quad\quad weight \leftarrow$ weight of $minAnswer$
11: $\quad\quad$ **if** $weight < leastWeight$ **then**
12: $\quad\quad\quad leastWeight \leftarrow weight$
13: $\quad\quad\quad bestMinAnswer \leftarrow minAnswer$
14: **return** $bestMinAnswer$

Since in each element $CovKeywords_i$ of the Cartesian product, each inclusion node has a unique keyword, the keyword will remain unique in the $Answer$ generated by $FindBestAnswerCovConstraint$ because the nodes containing that keyword will not be added to the $Answer$. Hence, the inclusion nodes in the $Answer$ cannot be removed when converting the $Answer$ to the $minAnswer$. Therefore, Algorithm 6 does not violate the inclusion constraint. In addition, since all possible combinations of the unique keywords of the nodes in the inclusion set are evaluated, no answer is missed.

The time complexity of the algorithm is $O((\prod_{i=1}^{s} |K_i|) \times l^2 \times |D_{max}|^2)$, where s is the

TABLE 6
Keywords used in DBLP data set.

Frequency	Keywords
0.0003	distance, discovery, scalable, protocols
0.0006	graph, routing, space, scheme
0.0009	fuzzy, optimization, development, support, environment, database
0.0012	modeling, logic, dynamic, application
0.0015	control, web, parallel, algorithms

average number of nodes in an inclusion set and $|K_i|$ is the number of unique input keywords in the ith inclusion node. Note that $\sum_{i=1}^{s} |K_i| \leq l-1$, where l is the number of input keywords. When the number of input keywords is small, the maximum cardinality of the Cartesian product is small. For example, for six keywords, the worst case happens when the inclusion set contains two nodes, one containing 3 unique keywords and the other containing 2 unique keywords. In this case, $\prod_{i=1}^{2} K_i = |K_1| \times |K_2| = 6$. Similarly, when l = 3, 4, 5 or 7, $\prod_{i=1}^{s} K_i$ is at most 1, 2, 4, or 8, respectively. Thus, since the number of query keywords is usually small in practice, Algorithm 6 is fixed-parameter tractable (FPT) [18].

7 Experimental Results

We implemented all the algorithms presented above. In addition, for the purpose of comparison and showing that previous approaches produce duplicate and non-minimal answers, we implemented four algorithms in the literature: $Dynamic$ [2], $BLINKS$ [4], $Community$ [6], and r-clique [8]. All of the algorithms are implemented in Java. The experiments are conducted on an Intel(R) Core(TM) i7-2720QM 2.20GHz computer with 16GB of RAM.

7.1 Data Sets and Queries

Two real world data sets, DBLP and IMDb, are used in our experiments. The DBLP graph is produced from the DBLP XML data[7]. The dataset contains information about a collection of papers and their authors. It also contains the citation information among papers. *Papers* and *authors* are connected together using the *citation* and *authorship* relations. The numbers of tuples of the 4 relations *author*, *paper*, *authorship* and *citation* are 947K, 2,578K, 5,221K, and 112K respectively. Keyword search over a DBLP graph is useful to find, e.g., a set of papers related to an author that covers a list of topics. The papers found in this way are more likely related to each other via authors. We used two approaches for assigning weights to the edges of the graph. In the first approach, the weight of the edge between two nodes v and u is $(\log_2(1+v_{deg}) + \log_2(1+u_{deg}))/2$, where v_{deg} and u_{deg} are the degrees of nodes v and u respectively.

7. http://dblp.uni-trier.de/xml/

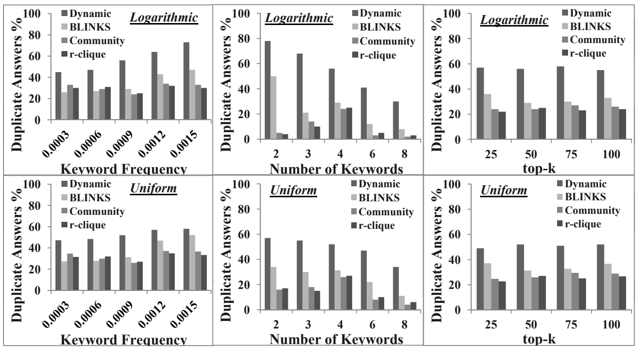


Fig. 4. Percentage of duplicate answers of different methods with different edge weights on DBLP dataset.

This approach is called $logarithmic$ edge weight and was used in [8], [6], [5], [2] . The second approach simply assigns the uniform weight of 1 to each edge. It is called $uniform$ edge weight and was used in [10]. The set of input keywords used in our experiments and their frequencies in the input DBLP graph are shown in Table 6. The queries used in our experiments are randomly generated from this set of keywords with the constraint that in each query all keywords have the same frequency (in order to better observe the relationship between run time and keyword frequency). Note that the input keywords shown in Table 6 were generated by the authors of [6] and used to generate queries in [6], [8]. We use the same set of input keywords and the same way to generate queries to make our results comparable to others.

The IMDb dataset contains the relations between movies and the users of the IMDb website that rate the movies[8]. The numbers of tuples of 3 relations *user*, *movie* and *rating* are 6.04K, 3.88K and 1,000.21K, respectively. The edges of the graph are weighted in the same way as for the DBLP graph. Due to the space limit, for the IMDb dataset we only present the results of query with keywords $house$, $king$, $night$, $city$, $city$, $world$ and $story$. The same set of input keywords is used in [6], [8].

7.2 Duplication of Previous Approaches

Our top-k method is guaranteed to generate duplication-free answers. In this section, we show the rates of duplicate answers of previous methods. Figure 4 shows the percentage of duplicate answers for four previous methods on the DBLP dataset with two different edge weights, different values of keyword frequency, different numbers of query keywords and different k values.[9] Two answers are

8. http://www.grouplens.org/node/73

9. Unless it is mentioned otherwise, in our results for DBLP, when not changing, the number of keywords is 4, keyword frequency is 0.0009 and top-50 answers are found. For the Community and r-clique methods, the r_{max} value is 8 and 5 for the $logarithmic$ and $uniform$ edge weights respectively.

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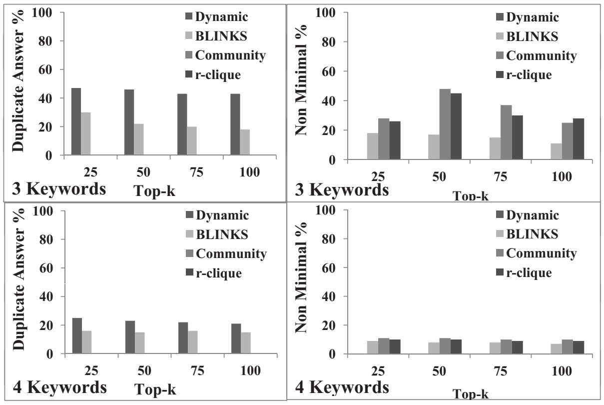

Fig. 5. Percentage of duplicate and non-minimal answers in different methods on IMDb dataset.

considered duplicates if they have the same set of content nodes. The rate of duplicate answers in the $Dynamic$ method [2] is higher than $BLINKS$ [4], $Community$ [6] and r-cliques [8]. This is because it finds minimum cost connected trees, and in most of the cases, the same set of content nodes are connected via different connections. $BLINKS$ also has a high rate of duplication. It is due to its policy of defining trees based on a unique root. The same set of content nodes may have a different root. The $Community$ and r-clique methods have the smallest rate of duplication among the existing methods because they divide the search space more wisely. But they still have some duplications. By increasing the frequency of keywords, the duplication rate of $Dynamics$ and $BLINKS$ increases. By increasing the number of keywords, the duplication rate generally decreases for $Dynamic$ and $BLINKS$. Changing the value of k does not have a significant effect on the duplication rate. All these previous methods have duplications for any value of k in the top-k answers.

The percentage of duplicate answers for 4 different methods on the IMDb dataset is shown in Figure 5, in which the edge weights are $logarithmic$ and r_{max} is 11 for the $Community$ and r-clique methods. The $Community$ and r-clique methods do not produce any duplicate answer for the queries used due to small numbers of content nodes (e.g., only 23 nodes contain keyword $house$). In addition, for 5 and 6 keywords, the duplication rate of all methods is close to zero due to small numbers of content nodes.

7.3 Non-Minimality of Previous Approaches

Both the complete and incomplete approaches proposed in this paper are guaranteed to generate only $minAnswers$. In Figure 6 we show the rates of non-minimal answers of four previous methods on the DBLP dataset with two different edge weights. The rates of non-minimal answers in $Community$ and r-clique are higher than those of $BLINKS$

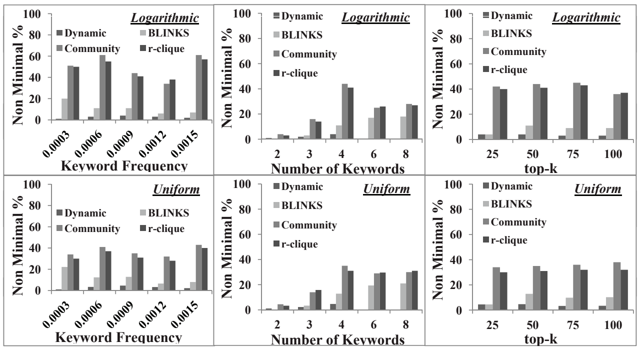

Fig. 6. Percentage of non-minimal answers of different methods with different edge weights on DBLP dataset.

and $Dynamic$. This is because for each keyword, $Community$ finds the closest keyword holder to the center of the community. However, the keyword may be covered by another node associated with the another keyword in the answer. This leads to non-minimal answers. The similar scenario occurs for the results of r-clique. In $Dynamic$, when merging two trees, their keywords cannot overlap. This leads to a very small rate of non-minimality. This is also valid on the IMDb dataset (Figure 5).

7.4 Run-time Comparison

One way to produce duplication-free answers is to post-process the answers generated from a keyword search method by removing the duplicates. In this section, we would like to see if our approach (which avoids generating duplicates) is faster than using the post-pruning method. Below we compare the run time of our methods to that of the r-clique and *Community* methods with the post-pruning procedure. When comparing with *Community*, we changed our Algorithm 2 to minimize the $centerDistance$ function which is the weight function used in *Community*. Note that minimize the $centerDistance$ function is slower than minimizing $sumDistance$. The modified Alg. 2 and the definition of $centerDistance$ are given in [13]. Note that the $centerDistance$ weight function is also used in $BLINKS$. We do not compare with $Dynamic$ because $Dynamic$ is too slow for its results to be put into the same graph with others. We do not directly compare with the original $BLINKS$ algorithm because $BLINKS$ generates much fewer answers than others. That is, if we allow all the methods to generate all the possible answers, $BLINKS$ only generates a subset of them while ours generates them all (i.e., $BLINK$ misses some answers.[10]) Thus, due to the

10. This is due to its use of distinct root semantic for producing answers. The number of answers produced by $BLINKS$ is $O(n)$ where n is the number of nodes in the graph. However, the number of answers in our model is $O(|D_{max}|^l)$ where $|D_{max}|$ is the maximum size of D_i for $1 \leq i \leq l$ and l is the number of query keywords. See [13] for an example on the incompleteness of $BLINKS$.

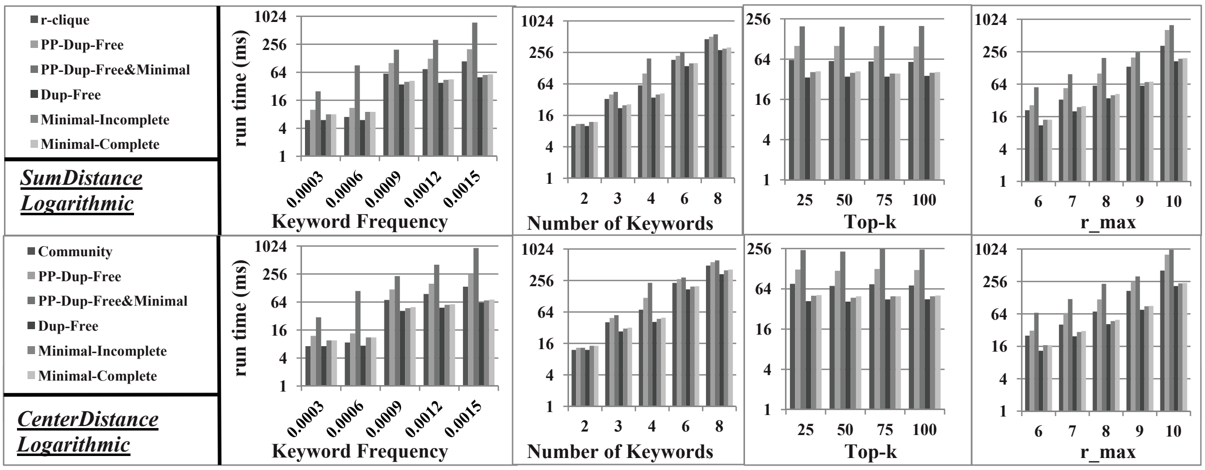

Fig. 7. Run time of different methods with different edge weights and two proximity functions on DBLP dataset. r_{max} is a distance threshold used in r-clique and $Community$.

incompleteness of $BLINKS$, we do not compare with the post-pruning version of original $BLINKS$, but its weight function is used in the *Community* method to compare with our approach with a modified Alg. 2 that minimizes *BLINKS*′ weight function.

Figure 7 shows the run time of different methods on DBLP with the $logarithmic$ edge weight. The first method is r-clique (or $Community$ in the second chart) which may generate duplicate and non-minimal answers. *PP-Dup-Free* refers to the r-clique (or $Community$ in the second chart) method that post-prunes duplicate answers. *PP-Dup-Free&Minimal* refers to the r-clique (or $Community$ in the second chart) method that post-prunes both duplicate and non-minimal answers. *Dup-Free* refers to our procedure for finding duplication free answers (i.e., Algorithms 1 and 2). The last two methods refers to our two approaches for finding duplication-free and minimal answers: the incomplete and complete approaches. To make fair comparisons, all of the methods use the same indexing method described in [8]. All the run times are the average time for producing one answer and presented in the logarithmic scale.

The run time of r-clique and $Community$ are slower than our duplication free method (*Dup-Free*) for all the three different settings. Sine they both use the same proximity measure, it seems to be a surprise. However, it is due to the fact that our method divides the search space into sub-spaces more wisely. The number of subspaces is usually smaller in our method. For example, for four keywords, assume that the best answer A contains only two nodes. The r-clique and $Community$ methods divides the search space into four subspaces (equal to the number of keywords). But our procedure divides the search space into two subspaces (equal to the number of nodes in A). Since the number of nodes is always no larger than the number of keywords, we gain better performance.

The results show that finding duplication free answers with post-processing is two to four times slower than our procedure. Finding duplication free and minimal answers using post processing is three to ten times slower than each of our approaches. By increasing the frequency of keywords, the number of keywords or the value of r_{max}, the run time increases. In addition, the run time (for producing one answer) does not change when the value of k changes. It shows that they all scale well with any number of required answers. Due to the space limit, the run time comparison on DBLP with the $uniform$ edge weights is omitted, but is given in [13].

7.5 Incomplete vs. Complete Approaches

The incomplete approach is faster in theory but it may miss some answers. On the other hand, the complete approach can produce all answers, but is slower. Figure 7 shows that for up to 6 keywords, the run time difference between the two approaches is less than 5% (which may be hard to see on the log scale in Figure 7). This is due to the small cardinality of the Cartesian product when the number of keywords is small and also because the worse case rarely happens in practice. For 7 to 10 keywords, our experiments show that the run time difference is up to 20%. In terms of missing answers, based on our experiments, the incomplete approach misses few answers for up to six keywords (less than 1% comparing to the complete approach). For 7 to 10 keywords, the incomplete approach misses up to 5% of the answers. Thus, the performances of the two approaches are close in practice.

7.6 The Quality of the Approximation Algorithm for Producing Minimal Answers

To evaluate the quality of the $minAnswer$ generated by the greedy Algorithm 4, we used exhaustive search to find the optimal (exact) answer that minimizes $sumDistance$. Figure 8 shows the average weight

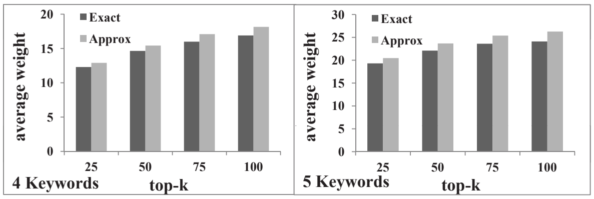

Fig. 8. Average $sumDistances$ of results from the exact and greedy algorithms for producing minimal answers on DBLP with $logarithmic$ edge weight. The r_{max} value is 8, keyword frequency is 0.0009 and the number of keywords is 4.

TABLE 7
Set of queries used in the user study.

Query	Keywords
1	parallel, graph, optimization, algorithm
2	dynamic, fuzzy, logic, algorithm
3	graph, optimization, modeling,
4	development, fuzzy, logic, control

of the answers produced by the exact and greedy algorithms for different values of k. The results shows that the difference of the two algorithms is at most 10% in practice, suggesting the high quality of the proposed greedy algorithm.

7.7 The Quality of the Minimal Answers

Finding duplication free answers is well motivated. Clearly, users prefer answers without duplication. However, it may be unclear whether users prefer minimal (more compact) over non-minimal (less compact) answers. To investigate this issue, we conducted a user study that compares *minimal* and *non-minimal* answers in terms of their relevancy to the query. For this purpose we used 4 meaningful queries for the DBLP dataset as shown in Table 7 and applied our Algorithms 1 and 2 to find duplication-free answers. We collected the first 10 non-minimal answers from the top-100 answers for each query, and used Algorithm 4 to convert them into minimal answers. We asked 8 users (who are graduate students in computer science but not involved with this work) to compare each pair of non-minimal and minimal answers by giving each answer a relevance score between 0 and 1 with 1 meaning completely relevant and 0 completely irrelevant to the query. Each answer is presented to the user as a Steiner tree generated using the first answer presentation method discussed in Section 3. Figure 9 shows a tree generated from a non-minimal answer for the first query (i.e. "parallel graph optimization algorithm") and Figure 10 shows the tree for its corresponding minimal answer.

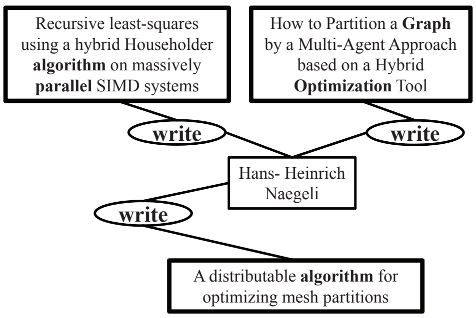

Fig. 9. A tree generated from a non-minimal answer.

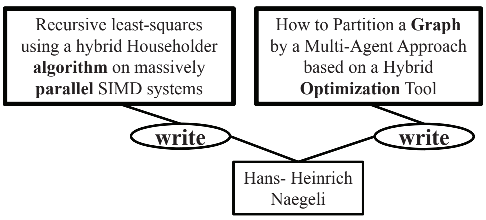

Fig. 10. A tree generated from a minimal answer.

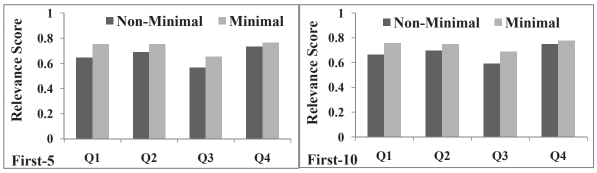

Fig. 11. Results of the user study.

For each answer we use the average of the relevance scores from the 8 users as the relevance score of the answer. For each query, we compute the average of the relevance scores of its first k non-minimal answers, and the average of the relevance scores of their corresponding minimal answers, where k = 5 or 10. These average relevance scores are presented in Figure 11. Clearly, *minimal* answers receive higher relevance scores than *non-minimal* ones in all the queries. This indicates that users prefer more compact answers as long as the set of nodes cover all of the query keywords. Also, larger answers have higher chance to include irrelevant nodes.

To further study the quality of the minimal answers, a state-of-the-art IR score is used to evaluate the answers. The IR scores are calculated based on the method used in [19]. The IR-score of a content node v for query Q is calculated as follows:

$$Score(v, Q) = \sum_{k \in Q \cap v} \frac{1 + \ln(1 + \ln(tf))}{(1 - s) + s\frac{cs}{AV_{cs}}} \times \ln \frac{N + 1}{df}$$

where, for a word k that appears in both v and Q, tf is the frequency of k in v, df is the number of nodes of the same type as v that contains k[11], cs is the size of v in characters, AV_{cs} is the average size of all of the nodes with the same type as v in characters, N is the total number of nodes with the same type as v and s is a constant. The same as in [19], we set s

11. For example, if v is a paper, df is the number of the papers containing keyword k in the dataset.

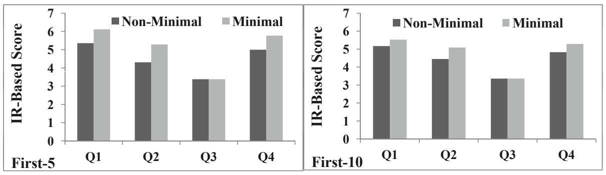

Fig. 12. Results of the IR-Based ranking.

to 0.2. Then, the combined score of the answer A that contains p content nodes is calculated as follows:

$$CombinedScore(A,Q) = \frac{\sum_{i=1}^{p} Score(v_i,Q)}{p}$$

The IR scores of minimal and non-minimal answers for the queries in Table 7 are presented in Figure 12. The result suggests that the IR scores of the minimal answers are generally higher than the non-minimal answers (except for the third query in which the IR-scores of both of the answer sets are very close.).

8 Conclusion

We have proposed novel and efficient methods for keyword search in graphs. A problem with existing approaches is that they may produce duplicate answers that have the same set of content nodes with trivial differences in their connections. To address this problem, we introduced a procedure that produces duplication free answers by wisely dividing the search space. In addition, since users are usually interested in exploring more compact answers [8] and in some applications (such as textbook selection) answers with unique contributions from each node are preferred, we defined *minimal answers* and proposed two algorithms for converting an answer to a minimal answer and two approaches to finding top-k or all duplication free and minimal answers. Our algorithms are guaranteed to generate duplication-free and minimal answers. We presented the rates of duplicate and non-minimal answers produced by previous approaches. We compared the run-time of our proposed methods to that of using post-pruning techniques to remove duplicate answers. We showed that our approaches are faster than post-pruning techniques. Finally, we show that the minimal answers have higher quality than non-minimal answers through a user study and a state-of-the-art IR weighting function.

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Mehdi Kargar received the BSc and MSc degrees in computer engineering from Sharif University of Technology. He is currently working toward the PhD degree in computer science at York University. He has published more than 20 papers in premier venues, including VLDB, SDM, ICDE, ICDM, CIKM, and PKDD. His research interests are in the area of database and data mining, focusing on keyword search in graphs, relational databases and social networks.

Aijun An received the PhD degree in computer science from the University of Regina in 1997. She is a professor in the Department of Computer Science and Engineering at York University. She held research positions at the University of Waterloo from 1997 to 2001. She joined York University in 2001. She has published widely in premier journals and conference proceedings. Her research interests include data mining, graph keyword search and social network analysis.

Xiaohui Yu received the BSc degree from Nanjing University, China, the MPhil degree from The Chinese University of Hong Kong, and the PhD degree from the University of Toronto, Canada. He has published more than 30 papers in premier venues, including SIGMOD, VLDB, ICDE, EDBT, SIGIR, and ICDM. He is an associate professor in the School of Information Technology, York University, Canada, and a member of the IEEE.