COSC 4111/5111 —Winter 2014

Posted: March 16, 2014
Due: April 7, 2014

Problem Set No. 3

This is not a course on *formal* recursion theory. Your proofs should be informal (but $\neq$ sloppy), correct, and informative (and if possible short). Please do not trade length for correctness or readability.

(1) **Without using Rice's theorem or lemma**, explore/prove

(a) the set $A = \{x : \text{ran}(\phi_x) \text{ has exactly five distinct elements}\}$ is not recursive. (I.e., "$x \in A$ is unsolvable"). Is it r.e.? **Why?**

Hint. Use as the "top" case function $rem(y, 5)$ which has a range of 5 elements.

(b) the set $D = \{x : \phi_x \text{ is the characteristic function of some set}\}$ is not recursive. Is it r.e.? **Why?**.

Hint. $D = \{x : \text{ran}(\phi_x) \subseteq \{0, 1\}\ \}$. Hmmm. Can you reuse the work we did with $\{x : \phi_x \text{ is a constant }\}$?

(c) the set $E = \{x : \text{ran}(\phi_x) \text{ contains only odd numbers}\}$ is not recursive. Is it r.e.? **Why?**

(2) Prove that there is a function $f \in \mathcal{P}$ such that $W_x \neq \emptyset$ implies $f(x)\downarrow$ and $f(x) \in W_x$.

Hint. To define $f(x)$ you want, given the verifier x (for W_x), to dovetail its computation as follows: consider systematically all pairs $\langle y, z\rangle$ *until* $T(x, y, z)$ holds. If so, set $f(x) = y$ (if not, go happily forever; this is the case $W_x = \emptyset$). *Make this mathematically precise*!

(3) Do Exercise 5.2.0.32, p.359.

(4) From Section 5.3 do Problem 23.