

A SIMPLE INDUCTION PROOF

Do Exercise 4.1.19 from the notes on logic.

Please make a good job both in answering the question, and uploading :)

This is not for credit either. Just practice!

Answer. To prove that $A \equiv B \vdash (\forall x)A \equiv (\forall x)B$ **without using the equivalence theorem.**

- 1) $A \equiv B$ $\langle \text{hyp} \rangle$
- 2) $A \rightarrow B$ $\langle 1 + \models_{\text{taut}} \rangle$
- 3) $B \rightarrow A$ $\langle 1 + \models_{\text{taut}} \rangle$
- 4) $(\forall x)A \rightarrow (\forall x)B$ $\langle 2 + \text{Ex. 4.1.16} \rangle$
- 5) $(\forall x)B \rightarrow (\forall x)A$ $\langle 3 + \text{Ex. 4.1.16} \rangle$
- 6) $(\forall x)A \equiv (\forall x)B$ $\langle 4 + 5 + \models_{\text{taut}} \rangle$

Instead of “ \models_{taut} ” in the three cases above I could have said “ping-pong”. □

BTW, I saw no submissions!