Lassonde School of Engineering

Dept. of EECS
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EECS 1028 Z. Practice Problem Set —Prep. for Exam; NOT for Submission or Credit

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On April 9, 2024, at 2pm I will post solutions:

1. Prove that $\mathbb{U} \times \mathbb{U}$ is a proper class.

2. Prove that if B is finite and $A \subseteq B$, then A is also finite.

3. Prove that an enumerable set is infinite.

4. Let A be enumerable. Show how —given an enumeration of A **without repetitions**— you can construct a **NEW** enumeration where **EACH** $x \in A$ **is enumerated infinitely many times**.

5. Prove that $\vdash (\forall x)A \to (\exists x)A$.

6. Prove that $\vdash (\forall x)(A \to B) \to (\exists x)A \to (\exists x)B$.

7. Use simple induction to prove that $n + 10 < 3^n$, for $n \geq 3$.

8. Consider the statement (formula)

$$(\exists x)A(x) \to A(c) \tag{1}$$

where c is a *new* constant, NOT found in $A(x)$.

Find now a *specific* **SIMPLE** example of $A(x)$ over the set $\mathbb{N}$ and choose a specific value of $c \in \mathbb{N}$ so that (1) becomes **false**, and **Therefore** we **cannot** prove (1), since proofs start from true axioms and preserve truth at every step.

9. Define the closure $\mathrm{Cl}(\mathcal{I}, \mathcal{O})$ by the specifications

(a) $\mathcal{I} = \{2\}$

(b) The ONLY operation in $\mathcal{O}$ is

$$(x, y) \mapsto x + y$$

That is, if the operation gets input x and y it produces output $x + y$.

Prove by induction on $\mathrm{Cl}(\mathcal{I}, \mathcal{O})$ that all its members are even natural numbers.

10. Using Simple Induction (SI) prove that $1^3 + 2^3 + \ldots + n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$, for $n \geq 1$.