MATH 1090.03D

Fall 2000

Date: Sep. 20, 2000

Due: Sep. 27, 2000—►**At the beginning of class**◄

Problem Set No. 1—On Chapter 2 of "GS".

NOTE. Whenever needed, BE-operator precedences **must** be **as given in class**. In particular, all associativities are **right**.

(0) Suppose that, for some given formula A, you want to find out whether or not $\neg A \vee A$ is a tautology, and you are going to use a truth table. Suppose that A has 11 Boolean variables. How many rows will your truth table have? **Why?**

(1) In each **row** below compare all formulas for pairwise **tautological equivalence**:

$$((p \vee q) \vee r), \quad (p \vee (q \vee r))$$

$$((p \wedge q) \wedge r), \quad (p \wedge (q \wedge r))$$

$$(((p \equiv q) \equiv r) \equiv p'), \quad (p \equiv (q \equiv (r \equiv p'))), \quad ((p \equiv (q \equiv r)) \equiv p'), \quad (p \equiv ((q \equiv r) \equiv p'))$$

(2) (Meta)prove that a formula A is a tautology iff $\neg A$ is unsatisfiable (a contradiction).

NOTE. Recall that "iff" means that there are two directions to do.

(3) (Meta)prove that for *any* two formulas P and Q, $\models (((P \Rightarrow Q) \Rightarrow P) \Rightarrow P)$.

(4) Express the following Boolean expression $A(p,q,r)$—given by a truth table—in **each of**:

(i) *Conjunctive Normal Form*, and

(ii) *Disjunctive Normal Form.*

NOTE. You may use "engineering notation", if you wish.

p	q	r	$A(p,q,r)$
t	t	t	f
t	t	f	f
t	f	t	f
t	f	f	f
f	t	t	f
f	t	f	t
f	f	t	t
f	f	f	f

(5) (Meta)prove that for any formulas A, B, C and D,

$$A, B, C \models D \text{ iff } \models A \Rightarrow B \Rightarrow C \Rightarrow D$$