MATH 1090.03D

Fall 2000

▶ Date posted: Dec. 5, 2000

▶ Due: **TBA by Web announcement immediately following the end of the CUPE strike.**

Alternative Problem Set No. 5—On Chapter 9 of "GS". See also our "Basic Equational Logic" report (on the Web).

This problem set is an **alternative** set to the one collected on Dec. 5, 2000 (for which solutions were posted on the same day).

It is **ONLY FOR THOSE students who exercised their right of choice** not to cross the picket lines, and therefore did not attend classes during the strike.

Only **ONE** problem set #5 will be accepted. Please **ignore this problem set IFF you have already handed in the original problem set #5.**

In the following problems you can use **any** tools that we have (e.g., Calculational/Equational proofs, Modus Ponens, Generalization/Specialization, Hilbert-style proofs, Post's Theorem, Deduction Theorem, Proof by contradiction, Monotonicity, etc.). Before you start a proof, think about the problem and choose the most convenient approach.

You should remember (and use, when appropriate) the following fact from class:

$$A \equiv B \vdash A \Rightarrow B \text{ and } A \equiv B \vdash B \Rightarrow A$$

and

$$A \Rightarrow B, B \Rightarrow A \vdash A \equiv B$$

which means that to prove $\Gamma \vdash A \equiv B$ you can do so by proving two things: $\Gamma \vdash A \Rightarrow B$ and $\Gamma \vdash B \Rightarrow A$.

- Do the following problems from the text, Chapter 9.

- 9.18, 9.20, 9.22, 9.23, 9.25, 9.27.

Also prove:

- $$\vdash (\forall x)A \wedge (\forall y)B \equiv (\forall x)(\forall y)(A \wedge B)$$

provided x is not free in B and y is not free in A

- Let P be a predicate of arity 1 and Q a predicate of arity 2. To which axiom groups, if any, do each of the following formulas belong?

$$(\forall y)\Big((\forall x)(P(x) \Rightarrow P(x)) \Rightarrow (P(c) \Rightarrow P(c))\Big)$$

$$(\forall x)(\exists y)Q(x,y) \Rightarrow (\exists y)Q(y,y)$$

- Let P be a predicate of arity 1. Prove or disprove:

$$\vdash \Bigg(\Big((\forall x)P(x) \Rightarrow (\forall y)P(y)\Big) \Rightarrow P(z)\Bigg) \Rightarrow \Bigg((\forall x)P(x) \Rightarrow \Big((\forall y)P(y) \Rightarrow P(z)\Big)\Bigg)$$

- Prove:

$$\vdash (\forall x)(A \Rightarrow B) \wedge (\exists x)A \Rightarrow (\exists x)B$$