York University

Faculties of Science and Engineering, Arts, Atkinson
MATH 1090. Problem Set #3
Posted November 4, 2005

Due: November 18, 2005; 4:00pm, in the course box

Section A

Worth reproducing (from the course outline):

"The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, nevertheless, at the end of all this consultation each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

See http://www.yorku.ca/secretariat/legislation/senate/acadhone.htm to familiarise yourselves with Senate's expectations regarding Academic Honesty.

The concept of late assignments does not exist."

In what follows, "prove $\vdash A$" means give a proof of A in *any* of the styles we have learnt —Hilbert, Equational, Resolution, by-Post, etc. unless a particular methodology is requested. Corresponding comment holds for "prove $\Gamma \vdash A$": Prove A from assumptions Γ.

(5 MARKS/Each) **Do the following problems**.

Important. If in any problem where you use a technique different than the one requested, then your maximum points will be 2.

Appropriate annotation is always required!

1. Use Resolution (in combination with the Deduction Theorem) —but NOT Post's theorem— to prove $\vdash A \vee (B \wedge C) \Rightarrow A \vee B$.

2. Use Resolution (in combination with the Deduction Theorem) —but NOT Post's theorem— to prove $\vdash (A \Rightarrow B) \Rightarrow (A \Rightarrow C) \Rightarrow (A \Rightarrow B \wedge C)$.

3. Use Resolution (in combination with the Deduction Theorem) —but NOT Post's theorem— to prove $\vdash (p \vee q \vee r) \wedge (p \Rightarrow p') \wedge (q \Rightarrow p') \wedge (r \Rightarrow p') \Rightarrow p'$.

4. We are in Boolean Logic.

Suppose that Γ is a set of assumptions, and A, B are two formulae.

We know that if $\Gamma \vdash A \wedge B$, then $\Gamma \vdash A$ and $\Gamma \vdash B$.

Is it also true that if $\Gamma \vdash A \vee B$, then $\Gamma \vdash A$ or $\Gamma \vdash B$?

If yes, then give a (meta)proof for any Γ, A, B.

If no, then use soundness to give a definitive counterexample for appropriately chosen Γ, A, B.

5. Prove by induction on the complexity of A that if x is not free in A then, for any term t, $A[x := t]$ is A.

6. Prove by induction on the complexity of A that $A[x := x]$ is the same string as A.

7. Prove $\vdash x = y \wedge y = z \Rightarrow x = z$.

8. Prove $(\forall x)(A \Rightarrow B) \Rightarrow (\exists x)A \Rightarrow (\exists x)B$.

9. Do 9.5 (p.174) from Gries/Schneider.

You must translate to standard notation before you start your proof.

10. Do 9.7 (p.174) from Gries/Schneider.

You must translate to standard notation before you start your proof.

11. Do 9.8 (p.174) from Gries/Schneider.

You must translate to standard notation before you start your proof.