## Lassonde Faculty of Engineering EECS

MATH1090. Problem Set No1 Posted: Sept. 21, 2015

Due: Oct. 6, 2015, by 2:00pm; in the course assignment box.



It is worth remembering (from the course outline):

The homework must be each individual's <u>own work</u>. While consultations with the <u>instructor</u>, <u>tutor</u>, and <u>among students</u>, are part of the <u>learning process</u> and are encouraged, nevertheless, at the end of all this consultation each student will have to produce an <u>individual report</u> rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.



- 1. (3 MARKS) Prove that no wff ends with the symbol "\".
  - *Hint.* Analyse formula-calculations (best approach), or use induction on formulas.
- **2.** (3 MARKS) Prove that  $\vee\vee$  is *not* a substring of *any* wff.
  - *Hint.* Analyse formula-calculations, or use exercise 1 (the latter approach is acceptable even if you did not answer 1).
- **3.** (1 MARK) Prove that  $((\neg(p \lor q)) \to p)$  is a wff.
- **4.** (6 MARKS) Recall that a schema is a tautology iff *all* its *instances* are tautologies.

Which of the following six schemata are tautologies? Show the whole process that led to your answers, including truth tables or equivalent short cuts, and words of explanation.

I note that in the six sub-questions below I am *not* using all the formally necessary brackets.

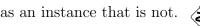
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Therefore be mindful of connective priorities and associativities!



- $((A \to B) \to A) \to A$
- $A \wedge B \rightarrow A \vee B$
- $A \vee B \rightarrow A \wedge B$
- $A \rightarrow B \equiv \neg B \rightarrow \neg A$
- $A \wedge (B \equiv C) \equiv A \wedge B \equiv A \wedge C$
- $A \lor (B \equiv C) \equiv A \lor B \equiv A \lor C$



- Recall that a schema is *not* a tautology iff it has an instance that is not.
- **5.** (3 MARKS) Prove that if for some formulae A and B it is the case that  $A, B \models_{\text{taut}} \bot$ , then it is also the case that  $\models_{\text{taut}} B \to \neg A$ . Here, using truth tables or truth-table tricks, you will show that if you have the left side of the "then" in the statement, then you must have the right side as well.
- **6.** (5 MARKS) By using truth tables, or using related shortcuts, examine whether or not the following tautological implications are correct.
- In order to show that a tautological implication that involves *meta*-variables for formulas—i.e., it is a schema— is *incorrect* you *must* consider a special case that is incorrect (since some other special cases might work).



Show the whole process that led to each of your answers.

- $p \models_{\text{taut}} p \land q$
- $A, B \models_{\text{taut}} A \wedge B$
- $A, A \rightarrow B \models_{\text{taut}} B$
- $B, A \rightarrow B \models_{\text{taut}} A$
- $p \wedge q \models_{\text{taut}} p$
- 7. (6 MARKS) Write down the most simplified result of the following substitutions, whenever the requested substitution makes sense. Whenever a requested substitution does not make sense, explain exactly why it does not.

Show the whole process that led to each of your answers in each case.

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Remember the priorities of the various connectives as well as that of the meta-expression " $[\mathbf{p}:=\ldots]$ "! The following formulae have not been written with all the formally required brackets.



- $q \lor (p \to p)[q := r]$
- $p \to \top[p := \mathbf{f}]$
- $(p \lor q)[p := \top]$
- $(\bot \to r \to q)[\bot := p]$
- $p \lor q \land r[q' := A]$  (where A is some formula)
- $p \lor (q \land r)[q := A]$  (where A is some formula)